Business Cycles when Consumers Learn by Shopping

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Abstract

Consumers rely on their shopping experiences to form beliefs about inflation. In other words, they *learn by shopping*. I study the consequences of this information friction for the transmission of macroeconomic shocks. I introduce learning by shopping in the benchmark New-Keynesian model and show that this friction anchors households' beliefs about inflation. However, the degree of anchoring is endogenous and depends on the model's structural features, including the monetary policy stance. Learning by shopping propagates the impact of demand shocks on output, even when prices are flexible. Price stickiness exacerbates this propagation, and the interaction of both frictions can be larger than the sum of the effects of each friction considered separately. Moreover, learning by shopping makes the slope of the Phillips curve a function of the degree of anchoring. For this reason, a more hawkish monetary policy can simultaneously anchor households' inflation expectations, flatten the Phillips curve, and lower the volatility and persistence of inflation. The model suggests that such a policy also has an unintended consequence: It makes the economy more vulnerable to exogenous shifts in aggregate demand.

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1 Introduction

Inflation plays a central role in macroeconomic theory and policy. Indicators like the Consumer Price Index (CPI) are the focus of attention of academics, central bankers, and market participants around the world. Given the resources devoted to control and forecast inflation, one would expect consumers would also be highly attentive to inflation developments.

But the empirical evidence suggests that most consumers do not pay attention to official inflation statistics: Their perception of current inflation differs substantially from the inflation reflected in the CPI or any other measure of aggregate inflation.[1] Moreover, providing households with information about those statistics has only partial and short-lived effects on their beliefs about inflation.[2] Instead of using public signals, the empirical evidence suggests that consumers rely on their own shopping experiences to form beliefs about inflation.[3] In other words, they *learn by shopping*.

In this paper, I investigate the consequences of learning by shopping (henceforth LBS) for the transmission of macroeconomic shocks. To do so, I introduce this information friction in a New-Keynesian model and use it to study analytically and quantitatively how this friction affects the transmission of aggregate shocks and the design of monetary policy.

Three results from this analysis stand out. First, LBS propagates and amplifies the impact of aggregate demand shocks on output, even when prices are flexible. Second, the interaction of this information friction with nominal rigidities in the price-setting behavior of firms amplifies the impact of demand shocks on output. Moreover, the propagation of demand shocks when both frictions are present can be larger than the sum of the effect of each friction considered in isolation. Finally, LBS makes the slope of the Phillips curve depend on the degree of anchoring of households' beliefs about inflation. But the degree of anchoring is itself a function of the strength with which the central bank responds to deviations of the inflation rate from its target. For this reason, a more hawkish monetary policy can simultaneously anchor households' inflation expectations, flatten the Phillips curve, and lower the volatility and persistence of inflation. Perhaps surprisingly, the model suggests that such a policy can also make the economy more vulnerable to exogenous shifts in aggregate demand.

[1] See Jonung (1981), Stanisławska (2019), Arioli et al. (2017), and Detmeister et al. (2016) for evidence from households in the E.U. and the U.S. Kumar et al. (2015), Coibion et al. (2018a) and Bryan et al. (2015) show that the beliefs about inflation from firms in New Zealand and the U.S. display similar behavior.

[2] See Cavallo et al. (2016) and Coibion et al. (2019).

[3] See, among others, Cavallo et al. (2017), Angelico and Di Giacomo (2019), Mosquera-Tarrío (2019), Coibion et al. (2020), D'Acunto et al. (2021b), and D'Acunto et al. (2021a).

Framework. I introduce LBS in a standard New-Keynesian model (hereafter NK model). The only modification to this well-known benchmark is the introduction of information frictions on the households' block of the economy. I assume households acquire information about prices when shopping for the different goods in their consumption bundle. Households use this information to form beliefs about inflation and make consumption, labor, and savings decisions conditional on those beliefs. Shopping experiences are idiosyncratic across households and provide them with a noisy signal about inflation. For this reason, their beliefs about inflation do not necessarily coincide with the inflation in posted prices, and they act as if they paid limited attention to aggregate inflation.

The noise in shopping experiences is a source of disagreement in beliefs. Each household shares a different view on the purchasing power of their income and the return of their financial assets. The heterogeneity in information translates into heterogeneity in consumption, labor supply, and asset holdings across households.

Despite this large heterogeneity, I show that the dynamics of aggregate output and inflation admit a simple characterization. The aggregate demand of this economy is described by a standard Euler equation augmented with the presence of an information wedge. This wedge captures the cross-sectional differences in households' beliefs about their permanent income and the present value of their financial wealth from the corresponding beliefs under full information. The aggregate supply side of this economy is described by a New-Keynesian Phillips Curve (NKPC) augmented with a second information wedge that captures the differences between the average real wage perceived by households and the real wage observed by firms.

The model shows that LBS simultaneously affects the economy's aggregate supply and demand blocks through two different information wedges. However, both wedges are the by-product of a single information friction, and their strength depends on the degree of anchoring of households' beliefs about inflation.

But the degree of anchoring (henceforth DOA) is an endogenous object in the model. The speed at which households incorporate news about inflation in their beliefs depends on the amount of information they can extract about this news from their shopping experiences. LBS makes households update their beliefs about current inflation slowly over time, making their beliefs about future inflation respond slowly to the aggregate shocks hitting the economy. In this sense, the LBS anchors households' inflation expectations.

I derive closed-form expressions for the equilibrium dynamics of aggregate output and inflation as a function of the DOA and use these expressions to characterize the existence and uniqueness of equilibrium in this model. I use these expressions to study how the DOA changes with the degree of nominal rigidities and the monetary policy stance. This allows me to study analytically the transmission of aggregate demand shocks in the model and its relationship with the monetary policy stance.

LBS propagates demand shocks. When prices are flexible,and agents in the economy have full information about inflation, shocks to nominal aggregate spending (like monetary policy shocks or preference shocks) do not generate comovement between inflation and output.

When consumers learn about inflation by shopping, this is no longer the case: Nominal shocks can have real effects, even when prices are flexible. The reason is that the labor supply decision of households depends on their perceived wages, not the real wage. To illustrate the consequences of this information wedge, consider a contractionary demand shock like the one observed during the Great Recession. When firms have flexible prices, this shock produces a fall in wages accompanied by a one-to-one reduction in prices by firms. When the inflation beliefs of households are anchored, they observe the fall in wages, but only part of the accompanying reduction in the aggregate price level. As a result, households perceive a lower real wage after the shock and reduce their consumption and labor supply in response.[4]

LBS allows inflation to comove with output following a demand shock. The information friction offers an alternative formalization of the original versions of the Phillips curve proposed by Phelps (1967) and Friedman (1968) to the one proposed by Lucas (1972).[5]

But compared to those theories, learning by shopping introduces a second channel through which the impact of the previous wedge is amplified. After the exogenous contraction in aggregate demand, households observe their nominal income fall. Since they only perceive part of the reduction in the aggregate price level, they confuse the shock with a drop in their permanent income. In response, they reduce their consumption further, amplifying the initial effect of the shock on aggregate output.

This amplification loop is reminiscent of the rational confusion theory introduced recently by Angeletos and Lian (2021). This paper shows how the same amplification channels can be obtained with a single information friction that simultaneously propagates the demand shock.

[4] I will argue in Section 6 that such a wedge in beliefs was indeed observed after the Great Recession.

[5] In particular, Friedman (1968), emphasized how the differences in perceptions about wages between firms and households is a source of comovement in output and inflation. Influenced by the work of Lucas (1972), he would expand this vision in his Nobel Prize lecture (Friedman (1977)) by attributing this wedge to information frictions by households. In the third section of this lecture, he notes: *"To workers, the situation is different: what matters to them is the purchasing power of wages not over the particular good they produce but over all goods in general. Roth they and their employers are likely to adjust more slowly their perception of prices in general - because it is more costly to acquire information about that - than their perception of the price of the particular good they produce. As a result, a rise in nominal wages may be perceived by workers as a rise in real wages and hence call forth an increased supply, at the same time that it is perceived by employers as a fall in real wages and hence calls forth an increased offer of jobs."*

Price stickiness amplifies the effects of LBS. The strong macroeconomic implications of LBS can look at odds with the observation that prices are rigid in the data. If prices don't change frequently, households' mistakes perceiving aggregate inflation can't be that large.

On the contrary, I show that price stickiness *amplifies* the effects of LBS. The intuition for this result is simple: Price stickiness gives rise to countercyclical markups, and this, in turn, makes real wages procyclical. To see the consequences of this observation, consider again a contractionary demand shock. Under sticky prices, this shock leads firms to increase their markups during a recession, producing a fall in real wages. This fall is the result of a reduction in the nominal wage that is more pronounced than the corresponding fall under flexible prices. The larger fall in nominal wages results in households perceiving an even larger reduction in real wages than firms, amplifying the information wedge that affects the labor market.

Surprisingly, the impact of a demand shock when both frictions are present can be larger than the sum of the corresponding impact when each friction is considered in isolation. The key behind this non-linearity is the endogenous nature of the DOA. An increase in the degree of price rigidities reduces the volatility of inflation, which in turn reduces the information contained in households' shopping experiences. As a result, higher price rigidity makes household beliefs about inflation more anchored, exacerbating the effect of the information wedges on the aggregate demand and supply of the economy.

To the best of my knowledge, this is the first paper introducing simultaneously nominal rigidities on firms and incomplete information about the aggregate inflation by households. The results show that this combination gives rise to powerful mechanisms that make shocks to aggregate demand the main drivers of the business cycle.

Monetary policy can flatten the slope of the Phillips curve. I show that, with LBS, monetary policy can flatten the slope of the NKPC by anchoring households' beliefs about inflation. As in standard models, the central bank can reduce the volatility of inflation by "flattening" the slope of aggregate demand with a more aggressive response to inflation. In this model, the lower inflation volatility reduces the information about aggregate shocks contained in households' shopping experiences. Consequently, the change in the policy stance increases the DOA of households beliefs about inflation. The model suggests that the central bank can indirectly affect the aggregate supply of the economy through its ability to anchor households' inflation beliefs. This result suggests that the under-reaction and persistence of households' beliefs about inflation observed in the data is a direct consequence of the success of monetary policy in stabilizing inflation. However, the result also suggests that the greater DOA also amplifies the impact of information frictions in the economy.

The previous mechanism also associates the flattening of the Phillips curve documented in the data to the more active monetary policy that followed Chairman's Volcker tenure at the Fed. The theory predicts that such a policy change flattens the NKPC by anchoring households' beliefs about inflation without changing its response to marginal costs. This prediction is consistent with the empirical evidence suggesting that there has been no change in the relationship between inflation and marginal costs (Del Negro et al. (2020), Barnichon and Mesters (2021), Hazell et al. (2020)). The literature has interpreted this finding as evidence in favor of the hypothesis that anchoring long-run inflation expectations has been the primary driver of the flattening of the NKPC. This paper suggests that households' inflation perceptions are the true force behind this empirical observation.

The theory also explains why the fit estimated NKPC's is better when households' expectations are used to estimate these curves (Coibion and Gorodnichenko (2015), Coibion et al. (2018b), Jorgensen et al. (2019)). There is a close relationship between inflation perceptions and expectations both in this theory and in the data.[6]This model suggests that, by incorporating households' inflation expectations, researchers are unintentionally controlling for the presence of the households' information wedge in the NKPC.

Extensions and quantitative analysis. I consider three extensions of the baseline model. First, I introduce shocks to aggregate technology shocks and show that LBS mitigates the real effects of these shocks. The intuition for this result is simple: A positive shock to aggregate TFP increases the real wage and the permanent income of households by reducing the aggregate price level. But the corresponding increase perceived by households is smaller in magnitude since LBS makes them learn slowly about the reduction in prices, hindering the transmission of TFP shocks on output.

Second, I extend the model to allow the noise in shopping experiences to be endogenous. I provide a microfoundation of learning by shopping as the product of households' optimal information acquisition about the aggregate price level. Households choose the attention allocated to aggregate inflation by trading the costs and benefits of acquiring information about this variable. Following the rational inattention literature pioneered by Sims (2003), I model the costs of acquiring information as a linear function of Shannon's mutual information. On the other hand, the costs of ignoring information are given by the welfare loss of making decisions that deviate from the full information benchmark. In turn, these costs are a function of the household's counterpart of the information wedges affecting the aggregate economy.

I show that for households, the information wedges due to learning by shopping have only second-order effects on their welfare. That is, inattention to aggregate prices can arise from small costs of acquiring information. This result is consistent with the observation by

[6]See, for instance, Jonung (1981), Armantier et al. (2016), Axelrod et al. (2018), Coibion et al. (2018a), and Candia et al. (2021).

Cochrane (1989) that the costs of deviating from the permanent income decision rule are arbitrarily small for a consumer. Like menu cost models, learning by shopping is a form of "near-rational behavior", as coined by Akerlof and Yellen (1985), where second-order individual losses can have first-order effects on the aggregate economy.

In the last extension, I introduce persistent learning in the model. The assumptions in the baseline model imply that households learn the true inflation rate at the end of the period. As a result, they have common knowledge about past variables, and learning lasts one period. I relax this assumption by allowing learning to be persistent over time. In this case, the model can no longer be solved analytically, and I propose a numerical method to solve it efficiently for a particular calibration.

I conclude the paper by studying the robustness of the earlier analytical results using a quantitative version of the model that includes the three aforementioned extensions. I calibrate the model to the U.S. data and show that LBS creates substantial propagation of demand shocks. In particular, a one standard deviation expansionary shock to aggregate demand produces an increase in output that is eight times larger than the corresponding impact under full information. Moreover, the presence of information frictions creates additional persistence on the response of output. Moreover, the information friction reduces the impact of technology shocks on output by a third.

Following Maćkowiak and Wiederholt (2015) and Afrouzi and Yang (2021), I also compare the predictions of a counterfactual "dovish" policy by the central bank with the behavior of macroeconomic variables observed in the Pre-Volcker era. The exercise shows that such a policy change can explain quantitatively the reduction in volatility and persistence in core CPI inflation observed in the post-Volcker era. It also explains the observed increase in the anchoring of households' inflation expectations in the Michigan Survey of Consumers. This counterfactual exercise suggests that the policy change to a more "hawkish" stance also had an unintended side-effect: It attenuated the impact of aggregate productivity shocks and contributed to making aggregate demand shocks the principal driver of the business cycle.

Outline. This paper contains seven sections, including this introduction. Section 2 reviews the related literature. Section 3 presents the model. Section 4 provides the main results of the paper. Section 5 extends the model to study the impact of technology shocks, and discusses the microfoundation of *learning by shopping* as the product of rational inattention to aggregate prices. Section 6 contains the quantitative analysis that shows the robustness of the earlier theoretical results, and compares them with the U.S. experience in the last decades. Section 7 concludes. The appendix contains all proofs and additional derivations, as well as the computational method used to solve the quantitative model of Section 6.

2 Related Literature

This paper contributes to the literature studying the role of information frictions on the transmission of aggregate shocks. The theory presented in this paper produces an information wedge in the labor market arising from differences in the real wage perceived by households and the the one perceived by firms. This channel was first proposed by Phelps (1967) and Friedman (1968) and was later formalized by Lucas (1972). Most of the subsequent literature focused on the role of information frictions in generating an upward sloping aggregate supply. The literature studied these frictions as an alternative to nominal rigidities like menu costs (e.g., Ball et al. (1988), Mankiw and Reis (2002) Mackowiak and Wiederholt (2009)). This paper contributes to this literature by providing a theory that allows both nominal rigidities and information frictions to coexist. It shows that these frictions are complements rather than substitutes and that their interaction amplifies the propagation of aggregate demand shocks.

Since Lucas (1973), most of the literature has focused on the role of information frictions on firms while abstracting from similar frictions on the consumer side. Notable exceptions include Reis (2006), Maćkowiak and Wiederholt (2015), and Gaballo and Paciello (2021). The first paper focused on the role of these frictions in explaining the behavior of consumption in the data. The second paper studied a problem with a more general inattention structure under flexible prices. Like this paper, Gaballo and Paciello (2021) present a theory where information frictions while shopping give rise to comovement between inflation and output in a static model. This paper complements their theory by emphasizing a different channel and showing that learning by shopping can also create comovement between inflation and output by generating a wedge in the labor market.

This paper is closely related to the work by Angeletos and Lian (2021). The aggregate Euler equation in this paper is similar to the one derived by those authors, with differences arising from the nature of the information friction and the fact that I allow learning to be persistent over time. The similarity is not surprising since both models feature information frictions that give rise to heterogeneity across households. However, the nature of the information friction studied in this paper is quite different. In their baseline model, there is no role for money, and intertemporal substitution in production propagates demand shocks. A second friction, rational confusion, amplifies the effect of the first friction. Here, the same friction is responsible for both the propagation and amplification of demand shocks.

This paper also contributes to the rational inattention literature pioneered by Sims (2003) and surveyed in Mackowiak et al. (Forthcoming). I contribute to this literature by studying the interaction between rational inattention and price stickiness and showing this interaction can have powerful amplification effects.

This paper contributes to the literature estimating the New-Keynesian Phillips Curve and the changes in its slope (Coibion and Gorodnichenko (2015), Coibion et al. (2018b), Del Negro et al. (2020), Hazell et al. (2020), Barnichon and Mesters (2021)). It provides a theoretical framework to interpret the recent evidence from this literature and provides an explanation for the better fit of specifications that take households' expectations into account.

Finally, the theory presented in this paper is motivated by the empirical literature documenting consumers' inattention to prices (surveyed by DellaVigna (2009), Anderson and Simester (2009), and Gabaix (2019)) and the impact of shopping and life experiences on their expectations about aggregate inflation (Malmendier and Nagel (2016), Cavallo et al. (2017), Angelico and Di Giacomo (2019), Coibion et al. (2019), Mosquera-Tarrío (2019) D'Acunto et al. (2021a)). This paper develops a model that allows studying the macroeconomic consequences of the frictions documented by this literature.

3 Learning by Shopping in a New Keynesian Model

In this section, I present an extension of the textbook New Keynesian model that introduces information frictions on the household block of the economy, giving rise to incomplete information about the aggregate inflation rate.

3.1 Model setup

Time is discrete and indexed by t. The model is inhabited by a continuum of households indexed by subscript $i \in [0, 1]$. Every household supplies labor and consumes an infinite variety of goods. Each consumption variety is produced by a different firm indexed by subscript $j \in [0, 1]$. The aggregate economy resembles the flexible price core underlying the New-Keynesian model. The only departure of this model is the introduction of information frictions on the household side, which I describe now.

Households. The problem of household i in period t is to maximize:

$$\mathrm{E}_{i,t} \sum_{k=0}^{\infty} \beta^k U \left(C_{i,t+k},\, N_{i,t+k};\, Z_{i,t+k} \right), \tag{1}$$

where $\beta \in (0, 1)$ is the households' discount factor, and $\mathrm{E}_{i,t}[\cdot] \equiv \mathbb{E}[\cdot | \mathcal{I}_{i,t}]$ denotes the expectation operator conditional on the information set of the household at the beginning of period t. This information set is denoted as $\mathcal{I}_{i,t}$ and is described below. The period utility function $U(\cdot)$ depends on the household's consumption index $C_{i,t}$, the labor supplied $N_{i,t}$, and a preference shifter $Z_{i,t}$. I assume that the per-period utility function $U(\cdot)$ takes the form

$$U\left(C_{i,t},\, N_{i,t};\, Z_{i,t}\right) = Z_{i,t}\left\{\frac{C_{i,t}^{1-\sigma}-1}{1-\sigma} - \frac{N_{i,t}^{1+\varphi}}{1+\varphi}\right\}, \tag{2}$$

and that households' consumption index is a CES bundle given by:

$$C_{i,t} = \left(\int_0^1 C_{i,j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3}$$

where $\varepsilon > 1$ denotes the elasticity of substitution across goods and $C_{i,j,t}$ denotes the consumption of variety j by household i . The preference shifter $Z_{i,t}$ captures exogenous shocks to household's discount rate and is given by

$$\begin{gathered}\log Z_{i,t} = \eta_t^{AD} + \xi_{i,t}^{z}, \\ \eta_t^{AD} \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_{AD}^2\right), \quad \xi_{i,t}^{z} \overset{iid}{\sim} \mathcal{N}\left(0, \zeta_x^2\right).\end{gathered} \tag{4}$$

The shock η_t^{AD} generates correlated desire across households to spend more in the current period and is the only source of exogenous shifts in aggregate demand in the economy. The shock $\xi_{i,t}^{z}$ produces idiosyncratic variations in the discount rate of each household and their only purpose is to limit their ability to observe the aggregate shock η_t^{AD} by observing their own shock $Z_{i,t}$. This point will be further discussed below.

The maximization of (1) is subject to the following sequence of budget constraints in every period:

$$\int_0^1 P_{j,t} C_{i,j,t} dj + B_{i,t} = Q_{i,t-1}^{-1} B_{i,t-1} + W_{i,t} N_{i,t} + D_{i,t}, \tag{5}$$

where $W_{i,t}$ denotes the nominal wage rate faced by household i, $P_{j,t}$ is the price of consumption variety j, $B_{i,t}$ denotes the quantity of nominally riskless one-period bonds purchased by this household in period t, $Q_{i,t}^{-1}$ is the gross nominal return of these bonds, and $D_{i,t}$ denotes the dividends the household receives from firm ownership.

Shopping and Paying. I introduce a restrictions on the timing of households decisions in the spirit of Rotemberg and Woodford (1997). Specifically, I divide the problem of each household into two consecutive stages: A *shopping stage* and a *paying stage*.

In the *shopping stage*, households choose the labor supply $N_{i,t}$ and order consumption varieties $C_{i,j,t}$, both of which are delivered in the following stage. During the *paying stage*, households receive the consumption varieties ordered in advance and supply labor. Households receive the dividends from firm ownership, observe the value of their expenditures $M_{i,t} \equiv \int_0^1 P_{j,t} C_{i,j,t} dj$, and adjust their bond holdings to make sure that the budget constraint binds.

Consequently, the problem of household i in period t is to choose the the labor supply

$N_{j,t}$ and the consumption of each variety $C_{i,j,t}$ to maximize (1) conditional on it's private information $\mathcal{I}_{i,t}$, subject to (5). At the end of every period, the household adjusts it's bond holdings $B_{i,t}$ to make sure that (5) binds.

Learning by Shopping. When households have full information about all prices, the *shopping* and *paying* assumption is inconsequential. I now introduce information frictions in the model by assuming that households do not observe $P_{j,t}$ directly. Instead, they receive a set of private noisy signals about prices at the beginning of each period, denoted as $S_{i,j,t}$. These signals are given by:

$$\log S_{i,j,t} = \log P_{j,t} + \epsilon_{i,t}, \quad \epsilon_{i,t} \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_\epsilon^2\right), \tag{6}$$

where $\epsilon_{i,t}$ is an idiosyncratic shock independent across households and time, and also uncorrelated with other shocks in the economy. Note that the noise is common across goods, for any given household.

This information structure relaxes the assumption that consumers have full information about the price index of the consumption bundle (3), denoted as

$$P_t \equiv \left(\int_0^1 P_{j,t}^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}.$$

The noise in signals $\epsilon_{i,t}$ limits the households' ability to infer P_t from their own shopping experiences. The idiosyncratic nature of shopping experiences produces dispersion in households' beliefs about the aggregate inflation rate.[7] Full information about P_t is nested as a special case when $\sigma_\epsilon^2 = 0$. To abbreviate, I refer to this information friction as *learning by shopping*.

In this model, the noise $\epsilon_{i,t}$ is microfounded as the result of Rational Inattention to aggregate inflation. The variance of noise in signals σ_ϵ^2 is endogenous and chosen by households to trade the costs and benefits of acquiring information about inflation.[8] For exposition purpose, I postpone the details of the Rational Inattention problem of households until Section 5, and assume until then that σ_ϵ^2 is constant and given.

There are two additional interpretations of the noise in signals $\epsilon_{i,t}$, which are not incom-

[7]The dispersion in beliefs about current inflation produces dispersion in the expectations about future inflation, which is a pervasive feature of the data. See Jonung (1981), Stanisławska (2019), Arioli et al. (2017), and Detmeister et al. (2016) for evidence from households in the E.U. and the U.S. Kumar et al. (2015), Coibion et al. (2018a) and Bryan et al. (2015) show that the beliefs about inflation from firms in New Zealand and the U.S. display similar behavior.

[8]Some readers may find too strong the assumption that consumers choose rationally the attention allocated to inflation. One can think of the signal structure 6 as a simple modeling shortcut to introduce in the model a pervasive feature of the data discussed in the introduction: Consumers' lack of knowledge about current inflation. In Section **??** I discuss how this noise can also be interpreted as a result of the heterogeneity in inflation rates at the household level or a specific form of bounded rationality.

patible with the interpretation that this noise comes from rational inattention to inflation.

A first interpretation is that this noise captures the heterogeneity in the inflation rates experienced by households. The empirical literature has documented a massive degree of heterogeneity in the inflation rates experienced by households.[9] In particular, Kaplan and Schulhofer-Wohl (2017) show that most of the variation in households' inflation rates comes from heterogeneity in the prices paid by households for the same type of goods. The authors show that variation in aggregate inflation explains only 9% of the variance of households' inflation rates over time. The distribution of inflation rates in the data moves in parallel with aggregate inflation, but the observable household characteristics have little power overall to predict household inflation rates. If some of this idiosyncratic heterogeneity is unobserved by the household at the point of purchasing goods, signals $S_{i,j,t}$ can be interpreted as the price paid by each household for a particular good and all the results in Section 4 will hold.

A second interpretation is that this signal captures a specific behavioral bias: Correlation neglect.[10] Even if households observe perfectly the price of each good in their consumption basket, inferring the behavior of aggregate inflation from the comovement among those prices is no easy task.[11] A household that ignores part of this correlation and chooses instead to interpret the movements as coming from relative prices would act as if he received noisy signals as in 6.

Households' information set. The information set of household i contains the history of wages, bond prices and preference shocks faced by the household. It also contains the history of private signals about each variety, as well as the total expenditures and dividends received in the previous period. Formally:

$$\mathcal{I}_{i,t} = \mathcal{I}_{i,t-1} \cup \{W_{i,t}, Q_{i,t}, Z_{i,t}\} \cup \{M_{i,t-1}, D_{i,t-1}\} \cup \left\{S_{i,j,t}\right\}_{j\in[0,1]}, \tag{7}$$

Before proceeding, it is important to highlight two properties of this information set.

First, the relative price $P^R_{j,t} \equiv P_{j,t}/P_t$ of each consumption variety is part of households' information set $\mathcal{I}_{i,t}$. To see this,notice that each household can construct a noisy signal of

[9] See Michael (1979), Hobijn and Lagakos (2005), Hobijn et al. (2009), and Kaplan and Schulhofer-Wohl (2017).

[10] See Enke and Zimmermann (2019) for evidence of this behavior in lab experiments and Kantorovitch (2020) for a recent application of this bias to explain misallocation of capital during booms.

[11] At the moment of writing this paper, there is a large debate among economists and policy commentators on whether the spike in the U.S. CPI inflation rate after occurring after the 2020 pandemic is a signal of a persistent increase in inflation. Commentators arguing that the spike is transitory have observed that a large part of this inflation is explained by a sharp increase in energy costs, which are highly volatile and depend on forces beyond the fundamentals of the U.S. economy. This debate shows that CPI inflation rate may not be an accurate measure of purchasing power, which is the object households are uncertain about in this paper. It also shows that professional economists, like the agents in this model, can have a hard time inferring movements in purchasing power from the comovement among prices.

the aggregate price by averaging across all its signals. Let

$$S_{j,t} \equiv \exp\left\{\int_0^1 \log S_{i,j,t} dj\right\}$$

denote this average signal. Using $S_{j,t}$, households can construct a second set of demeaned signals $S^R_{i,j,t} \equiv \log\left(S_{i,j,t}/S_{i,t}\right)$ that are exactly equal to the (log) relative price $P^R_{j,t}$. As a result, the information friction does not affect the relative value of goods that consumers perceive, only their perception of the aggregate price level.[12]

Second, households have complete information about *past* aggregate prices. In equilibrium, the total expenditures of each household $M_{i,t}$, depend on the price level P_t. At the end of the period, households observe their total expenditures, as well as all the variables in their income. They can then use their budget constraint (5) to infer P_{t-1} at the beginning of period t.[13]

Firms. Firms are price takers in the input market and use a linear technology of production $Y_{j,t} = A_t L_{j,t}$, where $L_{j,t}$ denotes the demand of labor by firm j in period t, and A_t is the aggregate productivity. This productivity is exogenous and given by:

$$\log A_t = \eta^{AS}_t; \quad \eta^{AS}_t \overset{iid}{\sim} \mathcal{N}\left(0, \sigma^2_{AS}\right). \tag{8}$$

The shock η^{AS}_t is common across firms and represents exogenous fluctuations in aggregate TFP, which are the only source of exogenous shifts in aggregate supply in the economy. The problem of firm j is to choose the price of its own variety $P_{j,t}$ to maximize the present value of its dividends, given by

$$\mathbb{E}_t \sum_{k=0}^{\infty} \Lambda_{t,k} \left(\frac{P_{j,t+k}}{P_{t+k}} - \frac{MC_{t+k}}{P_{t+k}}\right) \mathcal{C}_{j,t+k}, \tag{9}$$

where $MC_t \equiv W_t/A_t$ is the (nominal) marginal cost of producing variety j, $\mathcal{C}_{j,t} \equiv \int_0^1 C_{i,j,t} di$ is the demand for variety j across all households, and $\mathbb{E}_t\left[\cdot\right]$ denotes the full information expectation operator, and $\Lambda_{t,k}$ is a stochastic discount factor.

Every household in the economy has equal ownership of each firm, and their profits are redistributed accordingly. It follows that the stochastic discount factor used by every firm is an equally-weighted average of the stochastic discount factor of each household, which is given by:

$$\Lambda_{i,t,k} \equiv \beta^k \left(C_{i,t+k}/C_{i,t}\right)^{-\sigma} \left(Z_{i,t+k}/Z_{i,t}\right).$$

[12] See Gabaix (2014) for a model where inattention to prices alters the relative price perceived by households and the consequences of this form of bounded rationality.

[13] I relax this assumption in the quantitative model used in Section (6) to allow for slow learning of the inflation rate over time.

Finally, I assume that firms face nominal rigidities that prevent them from adjusting prices in every period. Specifically, I adopt the formalism proposed by Calvo (1983) and assume that each firm can reset its price only with probability $1-\theta$. This probability is exogenous, common across firms, and independent from the time elapsed since the last time the price was adjusted. It follows that a fraction θ of firms keep their prices unchanged in any period, and the average duration of a price is given by $\frac{1}{1-\theta}$.

Firms' information set. To isolate the role of *learning by shopping*, I assume that firms face no informational frictions. They can observe the value of aggregate productivity and their own marginal costs. Firms also understand that consumers form beliefs based on private signals. However, they don't observe these signals or the beliefs of each consumer directly. Consequently, they can't discriminate prices across customers or commit to holding a specific price.

Government. The central bank issues bonds $B_{i,t}$ at zero net supply and sets the interest rate $i_t \equiv -(\log Q_t - \log \beta)$ following a standard Taylor rule of the form:

$$i_t = \phi_\pi \pi_t, \tag{10}$$

where $\pi_t = \log P_t - \log P_{t-1}$ is the inflation rate measured from posted prices.

Auxiliary shocks. To conclude the description of the model, let $W_t \equiv \int_0^1 W_{i,t} di$ denote the average (nominal) wage of this economy. I assume that the wage of each household is given by $W_{i,t} = W_t \exp \xi^w_{i,t}$. Moreover, I assume that the bond price faced by each household is given by $Q_{i,t} = Q_t \exp \xi^q_{i,t}$. The shocks $\xi^w_{i,t} \overset{iid}{\sim} \mathcal{N}\left(0, \zeta_x^2\right)$, , $\xi^q_{i,t} \overset{iid}{\sim} \mathcal{N}\left(0, \zeta_x^2\right)$, and $\xi^z_{i,t}$ in (4) are independent across households and time and are also independent of other shocks.

This type of auxiliary shock is standard in the information frictions literature.[14] They can be alternatively microfounded as the result of market segmentation, idiosyncratic income risk, intermediation costs, perceptual noise, or rational inattention. For this paper, the actual microfoundation is not crucial. The only role of these shocks is to add noise to consumers' information set. This limits the information they can extract about aggregate inflation from sources different to their own shopping experiences.

Equilibrium definition. I focus on an equilibrium where agents hold rational expectations, make decisions contingent on their private information, and prices adjust to clear all markets.

Formally, a rational expectations equilibrium of this economy is defined by a set of stochastic processes for the average wage rate W_t, the price of each variety $\left\{P_{j,t}\right\}_{j\in[0,1]}$, the

[14] See, for instance, Lorenzoni (2009), Hellwig and Veldkamp (2009), Nimark (2014), Angeletos and Lian (2018), and Angeletos and Lian (2021).

labor supply and bond holdings of each household, $\{N_{i,t}, B_{i,t}\}_{i\in[0,1]}$, and the consumption of each variety by each household $\{C_{i,j,t}\}_{(i,j)\in[0,1]^2}$ such that:

1. Every household $i \in [0,1]$ maximizes its expected utility (1) conditional on its own information set (7) and budget constraint (5).

2. Every firm $j \in [0,1]$ maximizes the present value of its expected profits (9).

3. Agents have rational expectations in the sense that their *perceived law of motion* coincides with the *actual law of motion* of the economy.

4. The goods and labor markets clear.

By Walras law, the last condition also implies clearance of the asset market.

3.2 Equilibrium characterization

To keep the analysis tractable, I will work with a log-linear approximation of the model around a neighborhood of its non-stochastic steady-state with zero inflation. In what follows, I denote in lower case the log-deviation of a variable from its steady-state value. To keep the characterization of households' beliefs simple, I will also introduce the following assumption on the variance of the auxiliary shocks.

Assumption 1. *The variance of the auxiliary shocks ζ_x^2 is such that $\sigma_\epsilon^2/\zeta_x^2 \to 0$.*

This assumption guarantees that households rely exclusively on their shopping experiences to form beliefs about P_t. The assumption is not necessary for the characterization of the model, but it simplifies the exposition by isolating the role of *learning by shopping* in the transmission of macroeconomic shocks.[15]

Households first order conditions. I begin by solving the problem of each household. Let $\widehat{P}_{i,t} \equiv \mathrm{E}_{i,t}P_t$ denote the belief of household i about the aggregate price level, conditional on $\mathcal{I}_{i,t}$. The first order conditions of the problem of household i are:

$$C_{i,j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon} C_{i,t}, \tag{11}$$

$$N_{i,t}^{\varphi} C_{i,t}^{\sigma} = \mathcal{D}_{i,t}^{w} \frac{W_{i,t}}{\widehat{P}_{i,t}}, \tag{12}$$

[15]Consumers are Bayesian so their belief about P_t is a weighted average of their own shopping signal $S_{i,t}$, and other signals like their wage $W_{i,t}$. As $\zeta_A^2/\zeta_x^2 \to 0$, the weight assigned to signals other than $S_{i,t}$ shrinks to zero. Since ζ_x^2 is a free parameter in the model, we can always choose a region of the parameter space where this assumption holds approximately.

$$Q_{i,t} = \beta \mathrm{E}_{i,t}\left[\frac{\mathcal{D}^{w}_{i,t+1}}{\mathcal{D}^{w}_{i,t}}\left(\frac{C_{i,t+1}}{C_{i,t}}\right)^{-\sigma}\frac{Z_{i,t+1}}{Z_{i,t}}\frac{\widehat{P}_{i,t}}{\widehat{P}_{i,t+1}}\right]. \tag{13}$$

and the bond holdings are defined implicitly by the budget constraint (5). After a first-order approximation, we can express (12) and (13) as:

$$\varphi n_{i,t} + \sigma c_{i,t} = w_{i,t} - \widehat{p}_{i,t}, \tag{14}$$

$$c_{i,t} = \mathrm{E}_{i,t}c_{i,t+1} - \frac{1}{\sigma}\left(i_{i,t} - \widehat{\pi}_{i,t+1} + \mathrm{E}_{i,t}z_{i,t+1} - z_{i,t}\right), \tag{15}$$

where $\widehat{p}_{i,t} \equiv \mathrm{E}_{i,t}p_t$, and $\widehat{\pi}_{i,t+1} \equiv \mathrm{E}_{i,t}\pi_{t+1}$ denote household's i belief about the (log) price level and the inflation rate in $t+1$.

The details for the derivation of these expressions are available in Appendix B. Equation (11) is the standard demand under CES preferences, as expected from the observation that households have full information about the relative price. Equations (12) and (13) are similar to the textbook version of the labor supply and the Euler equation. The difference with respect to these counterparts is that households have incomplete information about P_t and condition their decisions to their own information set[16].

Firms first order conditions. Integrating (11) across consumers, we can express the aggregate demand for variety j as

$$\mathcal{C}_{j,t} \equiv \int_0^1 C_{i,j,t}dj = \left(P_{j,t}/P_t\right)^{-\varepsilon}C_t,$$

with $C_t \equiv \int_0^1 C_{i,t}di$. It follows that the problem of the firm in this setting is isomorphic to the problem of a firm in the standard model. From the first order conditions of this problem, we obtain the following New-Keynesian Phillips curve:[17]

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \lambda^{-1}rmc_t. \tag{16}$$

The parameter $\lambda \equiv \frac{\theta}{(1-\theta)(1-\beta\theta)}$ denotes the (inverse) slope of the curve with respect to real marginal costs, defined as $rmc_t \equiv w_t - p_t - a_t$. The parameter λ is increasing in the degree of price stickiness. Flexible prices are nested as the special case where $\theta = 0$, in which case $\lambda = 0$.

[16]Equations (12) and (13) feature a wedges, $\mathcal{D}^{w}_{i,t}$ that reflect distortions in expectations due to Jensen's inequality. Up to a first-order approximation, this wedges is equal to zero. See Appendix (B) for the details.

[17]The solution of this problem is well known (see, for instance, Chapter 3 in Galí (2015)), so I skip the details of the derivation of this curve.

Aggregate supply. Let $y_t \equiv \int_0^1 y_{j,t} dj$ and $n_t \equiv \int_0^1 n_{i,t} di$ denote, respectively, the aggregate output and labor supply of this economy. The production technology of firms implies:

$$y_t = a_t + n_t.$$

We can thus integrate (14) across households and use the market clearing condition $c_t = y_t$ to get

$$(\varphi + \sigma)\, y_t = w_t - p_t + \nu_t + \varphi a_t, \tag{17}$$

where

$$\nu_t \equiv \int_0^1 \{p_t - \widehat{p}_{i,t}\}\, di = p_t - \widehat{p}_t. \tag{18}$$

Here, the variable $\widehat{p}_t \equiv \int_0^1 \widehat{p}_{i,t} di$ denotes the average belief about the price level across households and ν_t is the average *perception error* about the price level. Equation (17) resembles the textbook aggregate labor supply of a model with full information, modified by the presence of ν_t. This term captures a first information wedge arising from *learning by shopping*, namely the wedge in the labor market. This wedge is driven by the differences between the average wage perceived by households and the real wage, which coincides with the wage perceived by firms.

Using (17) to replace the real marginal costs in (16), we arrive to the following expression characterizing the aggregate supply of this economy:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \alpha^*_{PC} \widetilde{y}_t - \lambda^{-1} \nu_t \tag{19}$$

where $\widetilde{y}_t \equiv y_t - ((1+\varphi) / (\sigma + \varphi))\, a_t$ denotes the deviation of output from its full information, flexible price counterpart, and $\alpha^*_{PC} \equiv (\sigma + \varphi)\, \lambda^{-1}$ is the slope of the Phillips curve in the full-information case.

In absence of information frictions ($\nu_t = 0$), the parameter α^*_{PC} serves as a sufficient statistic to characterize the comovement between output gap and inflation arising from aggregate demand shocks. The slope of aggregate supply in the economy is directly related to this parameter.

When households *learn by shopping*, a second term appears in the Phillips curve. In the spirit of Friedman (1977), this term captures the differences in the perception of real wages between firms and households. In this scenario, the parameter α^*_{PC} ceases to be a sufficient statistic since information frictions also induce positive comovement between inflation and the output gap. This, in turn, will change the slope of the aggregate supply schedule, as I will show below.

Aggregate demand. Derivation of the aggregate Euler equation of this economy is complicated by the fact that the Law of Iterated Expectations does not hold for the average expectations across households. Following Angeletos and Lian (2018) and Angeletos and

Lian (2021), we can use the budget constraint (5), together with households' Euler equation (15) to express the consumption of each household as a function of their expectations about current and future income and interest rates. Using this *beauty-contest* representation of individual consumption, we arrive to the following result.

Proposition 1. *(**Aggregate Euler Equation**) The aggregate demand is characterized by*

$$y_t = -\frac{1}{\sigma}\left(i_t - \mathbb{E}_t\pi_{t+1} + \mathbb{E}_t z_{t+1} - z_t\right) + \mathbb{E}_t y_{t+1} + \mathcal{X}_t + \beta\mathbb{E}_t\mathcal{X}_{t+1}, \tag{20}$$

where $\mathcal{X}_t \equiv \mathcal{H}_t + \mathcal{R}_t$ *is the sum of two information wedges given by*

$$\mathcal{H}_t \equiv \sum_{k=0}^{\infty}\beta^k \int_0^1 \left\{\mathrm{E}_{i,t}c_{t+k} - \mathbb{E}_t c_{t+k}\right\} di = \chi\mathbb{E}_t \sum_{k=0}^{\infty}\beta^k \nu_{t+k}, \tag{21}$$

and

$$\begin{aligned}\mathcal{R}_t &\equiv \sum_{k=0}^{\infty}\beta^k \int_0^1 \left\{\mathrm{E}_{i,t}r_{i,t+k} - \mathbb{E}_t r_{t+k}\right\} di \\ &= -\sigma^{-1}\mathbb{E}_t\left\{\nu^{\pi}_{t+1|t} + \sum_{k=1}^{\infty}\beta^k\left\{\nu^{\pi}_{t+k+1|t} - \phi_\pi \nu^{\pi}_{t+k|t}\right\}\right\}.\end{aligned} \tag{22}$$

Here, $r_{i,t+k} \equiv i_{i,t} - \pi_{t+1}$ *denotes the real interest rate faced by household* i, $\nu_t \equiv p_t - \widehat{p}_t$ *is the average perception error about the price level across households, and* $\nu^{\pi}_{t+k|t} \equiv \pi_{t+k} - \widehat{\pi}_{t+k|t}$ *is the average forecast error of inflation in* $t+k$ *across households. Finally,*

$$\chi \equiv \left(\frac{1-\beta}{\beta}\right)\left(\frac{\mathcal{M}\varphi}{\mathcal{M}\varphi + \sigma}\right).$$

Proof. See Appendix A.1. □

Equation (20) is similar to the textbook dynamic IS equation but is augmented by two information wedges that are the product of learning by shopping.[18]

The first wedge, $\mathcal{H}_t$, captures the effect of learning by shopping on households' perception of their *human wealth*, defined as the present value of the purchasing power of their wage and dividend income. Learning by shopping creates a wedge between households perceived human wealth, and their real wealth as measured by p_t. Equation (21) shows that this wedge is summarized by the cross-sectional differences between households' expectations of aggregate consumption and their full information counterpart. The differences, in turn, are proportional to the present value of average perception errors about the price level, ν_t, and its strength depends on the value of parameter χ.

[18] As discussed in Section ??, this representation is similar to the one derived by Angeletos and Lian (2021), with the main differences arising from the nature of the information friction and the fact that I allow learning to be persistent over time.

The second wedge, $\mathcal{R}_t$, captures the effect of learning by shopping on households' perception of their *non-human wealth*, defined as the present value of the real return of their assets. Learning by shopping makes households misperceive the current inflation rate. Equation (22) shows that this misperception creates a wedge in aggregate demand by generating dispersion on beliefs about current and future real returns. The magnitude of this wedge is proportional to the effect of inattention on households' forecasts about the future inflation rate, and also on their forecast about the nominal interest rate.[19]

Beliefs. Let $\widehat{p}_{i,t|s} \equiv \mathrm{E}_{i,s} p_{i,t}$ and $\widehat{\pi}_{i,t|s} \equiv \mathrm{E}_{i,s} \pi_{i,t}$ denote the beliefs of household i about the price level and inflation in t, given information available at s.[20] After observing signals (6), each household updates its beliefs about the aggregate price level using Bayes rule:

$$\widehat{p}_{i,t-h|t} = \widehat{p}_{i,t-h|t-1} + \kappa_h \left(s_{i,t} - \widehat{p}_{i,t|t-1} \right), \tag{23}$$

with $\kappa_h \in [0,1]$ denoting a Kalman gain that will be determined in equilibrium. Using the corresponding expressions for $h = 0$ and $h = 1$, we get

$$\widehat{\pi}_{i,t} = \widehat{\pi}_{t|t-1} + \kappa_0^\pi \pi_t + \kappa_0^\pi \left(p_{t-1} - \widehat{p}_{t|t-1} \right) + \kappa_0^\pi \epsilon_{i,t}, \tag{24}$$

with $\kappa_0^\pi \equiv \kappa_0 - \kappa_1$. Equation (23) shows that households update both their current and past beliefs after observing a signal in any period. Integrating (23) across households, we get:

$$\widehat{p}_{t-h|t} = \widehat{p}_{t-h|t-1} + \kappa_h \left(p_t - \widehat{p}_{t|t-1} \right), \tag{25}$$

with $\widehat{p}_{t|t-1} \equiv \int_0^1 \widehat{p}_{i,t|t-1} di$. Using this expression for $h = 0$ and $h = 1$, we can express the average perception error and the average inflation perception as:

$$\nu_t = (1 - \kappa_0) \left(p_t - \widehat{p}_{t|t-1} \right), \tag{26}$$

$$\widehat{\pi}_t = \widehat{\pi}_{t|t-1} + \kappa_0^\pi \pi_t + \kappa_0^\pi \left(p_{t-1} - \widehat{p}_{t|t-1} \right), \tag{27}$$

where $\widehat{\pi}_{t|t} \equiv \int_0^1 \widehat{\pi}_{i,t} di$ and $\kappa_0^\pi = \kappa_0 - \kappa_1$. Equation (29) shows how learning by shopping introduces inertia in the average beliefs of consumers about both current and future inflation.

Equilibrium computation The aggregate supply (17) and the aggregate demand (20) characterize the equilibrium behavior of inflation and output in this economy, conditional on households' beliefs about inflation. Equations (25), (26) and (27) show how households update their beliefs by using their past forecasts and signals acquired about the aggregate

[19] To see this why this is the case, note that households have rational expectations and the interest rate rule (10) is common knowledge across households. It follows that their forecasts of the interest rate are consistent with this rule, which depends only on the inflation rate.

[20] The notation above implies $\widehat{p}_t = \widehat{p}_{t|t}$.

price from their shopping experiences. To form beliefs about future prices, households need to know the law of motion of output and inflation, which is itself an equilibrium object of the model.

In general, this fixed point problem can only be solved numerically. However, the assumption that the exogenous shocks are i.i.d. and households have complete information about the past aggregate price level allows to characterize the equilibrium in closed form, as I show in the next section.

4 Learning by Shopping and the Transmission of Aggregate Shocks

In this section, I characterize the solution of the model and use it to show analytically the mechanisms through which learning by shopping affects the transmission of aggregate shocks. I start by characterizing the equilibrium degree of anchoring of households beliefs about inflation, and show how it changes with the structural parameters of the model. I then show how this endogenous anchoring propagates and amplifies the impact of demand shocks on output. Next, I show how price stickiness amplifies the effects of learning by shopping. I conclude by discussing how the monetary policy stance affects the slope of the Phillips curve in this model.

4.1 Equilibrium degree of anchoring

Beliefs about aggregate inflation. Since P_{t-1} is known at the beginning of each period, the beliefs about current inflation of household i are given by $\widehat{\pi}_{i,t} = \widehat{p}_{i,t} - p_{t-1}$. We can thus integrate (6) across varieties and subtract p_{t-1} from both sides to express the signal of household i about the inflation rate as

$$\pi^*_{i,t} = \pi_t + \epsilon_{i,t}, \quad \epsilon_{i,t} \overset{iid}{\sim} \mathcal{N}\left(0, \sigma^2_\epsilon\right), \tag{28}$$

with $\pi^*_{i,t} \equiv s_{i,t} - p_{t-1}$. To make further progress, I conjecture that, in equilibrium, the inflation rate is i.i.d. over time. We can then apply a well-known regression lemma for bivariate normal distributions to express household i posterior belief $\widehat{\pi}_{i,t}$ as:

$$\widehat{\pi}_{i,t} = \left(\frac{\text{Cov}_{i,t}\left[\pi^*_{i,t}, \pi_t\right]}{\text{Var}_{i,t}\left[\pi^*_{i,t}\right]}\right)\pi^*_{i,t} = \underbrace{\left(\frac{\text{Var}\left[\pi_t\right]}{\text{Var}\left[\pi_t\right] + \sigma^2_\epsilon}\right)}_{1-\psi_\pi}\left(\pi_t + \epsilon_{i,t}\right). \tag{29}$$

The parameter $\psi_\pi \in [0,1]$ measures the sensitivity of households' inflation perceptions to aggregate inflation. Under full information, we have $\sigma^2_\epsilon = 0$ and $\psi_\pi = 0$. In this case, inflation perceptions move one to one with the shocks hitting the aggregate inflation rate.

For a given variance of inflation, a higher variance of noise σ_ϵ^2 results in a larger value of ψ_π that approaches to one. The larger the value of ψ_π, the more inflation perceptions under-react to aggregate shocks hitting the inflation rate. Consequently, I interpret it as the degree of anchoring of inflation perceptions, and refer to ψ_π as the *degree of anchoring* of households beliefs.[21]

Using (29) together with the definition of the average perception error (18), and the fact that households have complete information about past aggregate prices, we get:

$$\nu_t = \nu_t^\pi = \pi_t - \widehat{\pi}_t = \psi_\pi \pi_t. \tag{30}$$

Equilibrium inflation and output. Equation (30) implies that $\mathbb{E}_t \nu_{t+1} = \mathbb{E}_t \nu_{t+1}^\pi = 0$. Consequently, the information wedges in (21) and (22) simplify to $\mathcal{R}_t = 0$ and $\mathcal{H}_t = \chi \nu_t$. The aggregate Euler equation (20) becomes

$$y_t = -\frac{1}{\sigma} \left(i_t - z_t \right) + \chi \nu_t.$$

Using (30) and the Taylor rule (10) to replace ν_t and i_t in this equation, we arrive to the following expression characterizing the aggregate demand of this economy:

$$\pi_t = -\alpha_{AD} \left(y_t - \widetilde{z}_t \right), \tag{31}$$

with

$$\alpha_{AD} \equiv \frac{1}{\sigma^{-1} \phi_\pi - \chi \psi_\pi} \tag{32}$$

with $\widetilde{z}_t \equiv \sigma^{-1} z_t$. As in the standard model with full information, the slope of aggregate demand α_{AD} depends on the response of the monetary authority to inflation, ϕ_π. With learning by shopping, the anchor ψ_π also enters this slope and its effect is proportional to parameter χ. When $\varphi = 0$, $\chi = 0$ and the effect of the information friction on the aggregate demand disappears.

Substituting ν_t in (17) in (16), we can write the NK Phillips curve of this model as

$$\pi_t = \widetilde{\beta} \mathbb{E}_t \pi_{t+1} + \alpha_{PC} \widetilde{y}_t, \tag{33}$$

where $\widetilde{\beta} \equiv \beta \lambda / \left(\lambda + \psi_\pi \right)$ and

$$\alpha_{PC} \equiv \frac{\lambda}{\lambda + \psi_\pi} \alpha_{PC}^* = \frac{\sigma + \varphi}{\lambda + \psi_\pi} \tag{34}$$

is the *slope of the NK Phillips curve* of the model. Notice how the degree of anchoring ψ_π

[21] This interpretation of anchoring is consistent with the definition used by Bernanke (2007), Mishkin (2007), Jorgensen et al. (2019), and Hazell et al. (2020) . Since a higher value of ψ_π implies that households have less knowledge of aggregate inflation, I will often refer to this term as the degree of *inattention* to aggregate inflation.

enters this slope. This will be important to explain the flattening of the Phillips curve, as discussed below.

Using the guess about the equilibrium process of inflation, we arrive to the following expression for the aggregate supply of this economy:

$$\pi_t = \alpha_{PC} \left(y_t - \widetilde{a}_t\right), \tag{35}$$

with $\widetilde{a}_t \equiv (1+\varphi) / (\sigma+\varphi) a_t$. Notice how the aggregate supply of this economy is not vertical as a result of both the anchor ψ_π, and the degree of price stickiness λ.

Equations (31) and (35) imply that equilibrium output and inflation are given by:

$$y_t = \Delta_y \widetilde{z}_t + \left(1-\Delta_y\right) \widetilde{a}_t, \tag{36}$$

$$\pi_t = \Delta_\pi \left(\widetilde{z}_t - \widetilde{a}_t\right), \tag{37}$$

where $\Delta_y \equiv \alpha_{AD} / \left(\alpha_{AD} + \alpha_{PC}\right)$ denotes the response of output to (normalized) aggregate demand shocks $\widetilde{z}_t$, and $\Delta_\pi = \alpha_{PC} \Delta_y$ is the response of inflation to the reduced form shock $u_t = \widetilde{z}_t - \widetilde{a}_t$.

Equation (37) verifies the conjecture that inflation follows an i.i.d. process. To complete the characterization of the equilibrium, all that is left is to find the equilibrium degree of anchoring ψ_π. The following proposition characterizes the existence of this value and gives a weak condition that guarantees its uniqueness.

Proposition 2. *(**Equilibrium degree of anchoring**) An equilibrium level for $\psi_\pi \in [0,1]$ exists and is given by the solution of*

$$1 - \psi_\pi = q \Delta_\pi^2 \psi_\pi, \tag{38}$$

with

$$\Delta_\pi \equiv \frac{\sigma+\varphi}{\lambda + (\sigma+\varphi)\sigma^{-1}\phi_\pi + (1-(\sigma+\varphi)\chi)\psi_\pi}, \tag{39}$$

and $q = \text{Var}[u_t] / \sigma_\epsilon^2$. Moreover, if $(\sigma+\varphi)\chi < 1$, this equilibrium is unique.

Proof. See Appendix A.2. □

Notice that, for reasonable values of parameters β, φ, and σ, the condition for existence is always met.

Equations (36), (37) and (38) characterize the equilibrium of this economy. I now use these equations to analyze the impact of inattention to aggregate inflation on the transmission of aggregate shocks a_t and z_t. To do so, it will be important to know how ψ_π changes with the structure of the economy. The following proposition shows that, the same condition that guarantees the uniqueness of the equilibrium level of inattention implies that ψ_π is increasing in the degree of price stickiness and the response of monetary policy to inflation.

Proposition 3. *(**Endogenous degree of anchoring**) If $(\sigma + \varphi)\chi < 1$ the anchor ψ_π is increasing in the degree of price stickiness λ, and the response of monetary policy to inflation ϕ_π.*

Proof. See Appendix A.3. □

The intuition behind this result is simple. An increase in nominal rigidities reduces the volatility of inflation. For a given level of noise in signals σ_ϵ^2, the lower volatility of inflation results in a reduction of the informativeness of the signals received by households. Consequently, they put less weight on these signals, as implied by (29).

I will now show that this observation has important implications for the transmission of aggregate shocks and the response of the economy to a change in the monetary policy stance.

4.2 The propagation of demand shocks

To isolate the effect of learning by shopping, start by considering the economy under flexible prices ($\lambda = 0$). The following proposition characterizes how learning by shopping allows the propagation of demand shocks even when prices are flexible.

Proposition 4. *(**Propagation of demand shocks**) The equilibrium response of output an inflation to aggregate demand shocks is given by:*

$$\frac{\partial y_t}{\partial z_t} = \sigma^{-1}\Delta_y > 0, \qquad \frac{\partial \pi_t}{\partial z_t} = \sigma^{-1}\Delta_\pi > 0,$$

with

$$\Delta_y \equiv \frac{\sigma^{-1}\psi_\pi}{(\sigma + \varphi)\,\sigma^{-1}\phi_\pi + (1 - (\sigma + \varphi)\,\chi)\,\psi_\pi} \tag{40}$$

and Δ_π defined in (39).

Proof. See Appendix A.4. □

The above proposition shows that learning by shopping allows for comovement between inflation and output as a result of demand shocks, and that the degree of this comovement is increasing in the anchor ψ_π. Under full information, this shock has no effects on output and is completely absorbed by the inflation rate.

This proposition, however, conflates two different channels that arise simultaneously from learning by shopping. The first channel propagates demand shocks by shifting the slope of the aggregate supply of the economy. The second channel amplifies the first channel by making aggregate demand steeper.

The aggregate supply channel. To understand the first channel, it is helpful to represent on a diagram the labor market of this economy, as given by equations (16) and (17) under flexible prices. The first diagram of Figure 1 shows the effect of the aggregate demand shock in the labor market of this economy.

Point ***A*** in the plot corresponds to the initial equilibrium before the shock. Suppose there is an unexpected contraction in the aggregate demand in the economy. Proposition 4 shows that, under full information ($\psi_\pi = 0$), this contraction is fully absorbed by the inflation rate. Wages and prices fall proportionally due to firms' desire to keep markups constant, and the real wage and the labor supplied by households remain unchanged. After the shock, the equilibrium remains at point ***A***.

Suppose now that households' inflation beliefs are anchored ($\psi_\pi > 0$). In this case, households observe the reduction in wages that follows the demand shock. But the inflation perceptions are anchored, so they observe only part of the fall in aggregate prices. As a result, households perceive a reduction in the real wage even though it remains constant after the shock. As illustrated in the first panel, the perception error $\nu_t \equiv p_t - \widehat{p}_t$ acts as wedge that shifts the labor supply and moves the economy to a new equilibrium with lower output at the same real wage, indicated by point **B**. While the real wage remains constant, prices and wages fall with the demand shock, but they fluctuate less compared to the full information case.

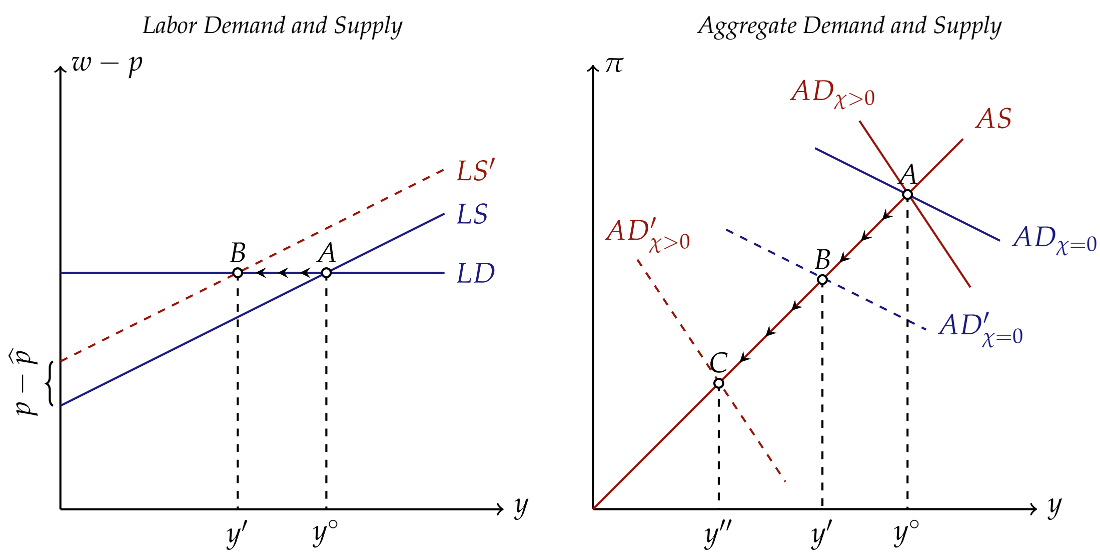


Figure 1: Propagation and amplification of a contractionary demand shock

Notes: The figure illustrates how *learning by shopping* propagates and amplifies the effect of a contractionary aggregate demand shock. The first panel shows how the differences in the perceived real wage between households and firms reduce the labor supply and output after the shock. The second panel shows how the intial effect is amplified by a fall in permanent income households' perceive.

The aggregate demand channel. The previous analysis of labor market offers an incomplete view of the total effect of *learning by shopping*. This information friction also affects households' perception of their human wealth. The perception error ν_t enters as a wedge in the aggregate Euler equation (20) and it's effect on aggregate demand is captured by the presence of parameter χ in equations (31) and (40).

To visualize the amplification coming from this channel, the second diagram of Figure 1 plots the aggregate demand and supply of this economy, as given by (31) and (35). The aggregate supply has a positive slope, as implied by the previous analysis of the labor market.

When $\chi = 0$, the aggregate demand is equal to its full information counterpart, as illustrated by the blue downward sloping line. In this case, the contractionary shock to aggregate demand shifts the AD curve and moves the equilibrium from point ***A*** to point ***B*** as a consequence of households' perceiveing an increase in real wages.

When $\chi > 0$, the aggregate demand becomes steeper compared to its full information counterpart, and aggregate demand becomes more sensitive to the shock. Equation (31) shows that this change also makes aggregate demand more sensitive to exogenous shifts in aggregate demand. As a result, the same shock now displaces further the aggregate demand curve. This additional amplification is the result of the fall in households perception of their permanent income: They observe the reduction in the the present value of their wage and dividend income after the shock, but only observe part of the reduction in the aggregate price level. In response, households reduce their consumption further, amplifying the initial effect of the shock. The new equilibrium, indicated by point **C** in the graph, features lower output and inflation than the case where the mechanism is muted.

4.3 The amplification of demand shocks

Equation (36) implies that the equilibrium response of output to an aggregate demand shock is given by

$$\frac{\partial y_t}{\partial z_t} = \Delta_y = \sigma^{-1} \left(\frac{\lambda + \psi}{\lambda + (\sigma + \varphi)\, \sigma^{-1} \phi_\pi + (1 - (\sigma + \varphi)\, \chi)\, \psi} \right), \tag{41}$$

Given the previous discussion, it is not surprising that demand shocks are further propagated when both nominal rigidities and information frictions from learning by shopping are present. Intuitively, both frictions affect the slope of aggregate supply, allowing demand shocks to have real effects in the economy.

Perhaps more surprising is that the impact of a demand shock when *both* frictions are present can be larger than the sum of corresponding impact when each friction is considered in isolation. The following proposition establishes the conditions under which amplification can arise.

Proposition 5. ***(The interaction between learning by shopping and sticky prices)*** *Let* $[\partial y_t/\partial z_t]^{SP}$ *denote the response of output to a demand shock when there is full-information and sticky prices* ($\sigma_\epsilon^2 > 0$, $\lambda = 0$)*. Let* $[\partial y_t/\partial z_t]^{LS}$ *denote the corresponding response when there is incomplete information and flexible prices* ($\sigma_\epsilon^2 > 0$, $\lambda = 0$)*. Let* $[\partial y_t/\partial z_t]^{LS+SP}$ *denote the response when there is both incomplete information and sticky prices* ($\sigma_\epsilon^2 > 0$, $\lambda > 0$)*. If*

$$\frac{\psi_\pi^{LS+SP}}{\psi_\pi^{LS}} - 1 > \left(\frac{\sigma}{\sigma+\varphi}\right)\frac{\lambda}{\phi_\pi} \tag{42}$$

and $(\sigma+\varphi)\,\chi < 1$*, then*

$$[\partial y_t/\partial z_t]^{LS+SP} > [\partial y_t/\partial z_t]^{LS} + [\partial y_t/\partial z_t]^{SP}$$

Proof. See Appendix A.5. □

To understand this result, it is important to recall that, while the equilibrium conditions for output and inflation are linear, the model is not. The anchor ψ_π is an endogenous object that changes with the degree of price stickiness.

Proposition 3 shows that ψ_π increases with the degree of price stickiness, and equation (41) shows that the response of output to demand shocks increases with the degree of anchoring of expectations. As a result, if the response of ψ_π to the introduction of sticky prices is strong enough, the new scenario will propagate demand shocks beyond what is possible by each friction considered independently.

4.4 The flattening of the Phillips curve

Proposition 3 shows that ψ_π depends on the response of the central bank to the inflation rate. Consider a change of monetary policy to a more "hawkish" stance, reflected as an increase in the value of ϕ_π. This policy change flattens the aggregate demand by making the interest rate more sensitive to variations in inflation. In absence of information frictions, this policy unambiguously reduces the volatility of inflation induced by demand shocks.

With learning by shopping, the reduced volatility in inflation increases the degree of anchoring ψ_π, which in turn increases the propagation of demand shocks. This can potentially mitigate the reduced volatility from the policy change, so the final effect on the volatility of output and inflation is ambiguous.

What is not ambiguous is the response of the aggregate supply to this policy change. The increase in anchoring produced by the higher value of ϕ_π reduces the slope of the aggregate supply of this economy, as the following proposition makes clear.

Proposition 6. *(**Monetary policy and the Phillips curve**) An increase in the response to inflation ϕ_π by the central bank flattens the slope of the Phillips curve. Formally,*

$$\frac{\partial \alpha^*_{PC}}{\partial \phi_\pi} = -(\sigma + \varphi)\left(\frac{1}{\lambda + \psi}\right)^2 \frac{\partial \psi_\pi}{\partial \phi_\pi} < 0$$

where α^*_{PC} is defined in (34).

Proof. See Appendix A.6. □

To understand the implications of this proposition, recall that the NK Phillips curve in this model is, in general, given by:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \alpha^*_{PC} \widetilde{y}_t - \lambda^{-1} \nu_t. \quad (43)$$

In absence of information frictions ($\nu_t = 0$), the parameter α^*_{PC} serves as a sufficient statistic to characterize the comovement between output and inflation arising from aggregate demand shocks. This parameter depends only on the degree of price-stickiness λ, which is independent of the monetary policy stance.

When households learn by shopping, a second term appears in the Phillips curve. This term captures the impact of the differences in perception of wages between households and firms. This information wedge acts as an endogenous source of fluctuations in the firms' desired markup. In this case, the parameter α^*_{PC} ceases to be a sufficient statistic of the slope of the aggregate supply since the information friction also induces positive comovement between inflation and the natural output gap. As shown in (19), the slope of aggregate supply is now a function of the degree of anchoring of households' inflation perceptions. Moreover, the degree of anchoring, and consequently, the slope of aggregate supply, is endogenous and depends on the monetary policy stance. A policy change will flatten the slope of the Phillips curve, even if λ remains constant.

We can use equation (43) and Proposition (A.6) to interpret several results on the empirical literature documenting a flattening of the slope of the Phillips curve.

Several researchers have observed that the correlation of inflation and different measures of the output gap has fallen over time, with the fall starting at some point in the 80's.[22] The timing of this flattening of the Phillips curve coincides with the change in the way monetary policy was conducted after Paul Volcker was appointed Chairman of the Board of Governors of the Federal Reserve System (Clarida et al. (2000)). In the model, such a change is captured by an increase of the central bank to the inflation rate, measured by ϕ_π.

[22] See, for instance, Ball and Mazumder (2011), Kiley (2015), Blanchard (2016), Stock and Watson (2019), Höynck (2020), and Barnichon and Mesters (2020).

Consider now an econometrician that estimates estimate α_{PC} using a measure of inflation π_t, the output gap $\widetilde{y}_t$ and some measure of expectations under full-information $\mathbb{E}_t\pi_{t+1}$ (for instance, the expectations of professional forecasters). The model predicts that this econometrician would be estimating a specification like (33), where the slope α_{PC} is endogenous and changes with the monetary policy stance. This econometrician will observe that, after the policy change, the slope α_{PC} has become flatter, consistent with the empirical evidence for the U.S..

Recent work by McLeay and Tenreyro (2019), Fitzgerald et al. (2020) and Hazell et al. (2020) has estimated the slope of the Phillips curve exploiting regional variation to control for the confounding effect of aggregate variables like long-run inflation expectations and the response of monetary policy to demand shocks. These authors find that the slope of the Phillips curve is small and has remained constant in the last decades. Their evidence is consistent with the finding that the response of U.S. inflation to variations in marginal costs has not changed over time (Del Negro et al. (2020), Barnichon and Mesters (2021)).

Equation (43) suggests that the estimation strategy of these authors controls for the effect of the the average perception error ν_t on inflation, delivering estimates of the full-information slope α^*_{PC} which is only a function of the degree of price stickiness λ. Moreover, the response of inflation to marginal costs is also a function λ only, as shown in (16). If the degree of price stickiness has not changed over time, the empirical findings of this literature are consistent with the NK Phillips curve in this model.

Summarizing, proposition 6 shows that the comovement between inflation and output can fall, even if the degree of price stickiness λ is constant, as a result of the anchoring of households' inflation perceptions. The previous observation offers a way to reconcile the conflicting evidence regarding the estimation of the Phillips curve.

To conclude, several authors have also estimated equations similar to (33) using different proxies for the expectations of economic agents (Coibion and Gorodnichenko (2015), Coibion et al. (2018b), Jorgensen et al. (2019)). A common finding in these exercises is that the expectations of households allow the estimated model to fit better the data.

This model offers a potential justification for the use of households inflation expectations when estimating Phillips curves in the data. Note that we can use (30) to rewrite (43) as:

$$\pi_t = \frac{\lambda}{1+\lambda}\beta\mathbb{E}_t\pi_{t+1} + \frac{\sigma+\varphi}{1+\lambda}\widetilde{y}_t + \frac{1}{1+\lambda}\widehat{\pi}_t. \tag{44}$$

As discussed in the introduction, the data suggests that there is a very close relationship between households' perceptions of current inflation, and their expectations about future inflation. If households answer expectations surveys by reporting their current perception[23], the addition of their expectations to econometric specification can act as a proxy

[23]This type of behavior is consistent with households perceiving that the 12 month inflation rate follows a

of the missing term $\widehat{\pi}_t$ in the right hand side of (44).[24]

Moreover, the beliefs of households are persistent over time, as a byproduct of learning by shopping. Consequently, the presence of $\widehat{\pi}_t$ (44) adds time-varying persistence to the behavior of inflation over time, which is a common feature in the data.[25]

Taken together, these observations may explain the common empirical finding that the Phillips curves fit better the data when the expectations of households are used in their estimation.

5 Extensions: Technology Shocks and Endogenous Information Acquisition

In the first part of this section I discuss the impact of TFP shocks when consumers learn by shopping. I show that the same forces that propagate and amplify demand shocks also attenanuate the aggregate impact of technology shocks on output. In the second part, I provide a microfoundation of learning by shopping as the result of households rational inattention to aggregate inflation, making the information acquired by households and endogenous function of the structural parameters of the model.

5.1 The attenuation of technology shocks

I now turn attention to the transmission of technology shocks. The following proposition characterizes how learning by shopping attenuates the effect of productivity shocks on output.

Proposition 7. *(**Attenuation of technology shocks**) Let $[\partial y_t/\partial a_t]^{FI}$ denote the equilibrium response of output to a technology shock under full information and let $[\partial y_t/\partial a_t]^{LS}$ denote the response when households' learn by shopping. We have:*

$$\left[\frac{\partial y_t}{\partial a_t}\right]^{LS} = \left(1-\Delta_y\right)\left(\frac{1+\varphi}{\sigma+\varphi}\right) \leq \left[\frac{\partial y_t}{\partial a_t}\right]^{FI},$$

where Δ_y defined in (40).

Proof. See Appendix A.7. □

random walk. As shown by Atkeson and Ohanian (2001) and Stock and Watson (2007), this is indeed a good approximation of the data generating process of this variable.

[24] While firms in this model have full information, the evidence suggests that their inflation expectations behave closely to those of households. If this is the case, this equation would depend on the expectations of households only.

[25] See Gali and Gertler (1999) and Gallegos (2021) for more recent evidence.

To understand this result, it is useful again to plot the labor supply and demand of this economy. The first diagram of Figure 2 shows the effect of a positive productivity shock on the labor market of this economy.

The equilibrium before the shock is highlighted by point ***A*.** The increase in aggregate productivity allows firms to produce at a lower (nominal) marginal cost. Firms want to keep their markups constant, so they reduce prices proportionally leading to an increase in the real wage. This effect is captured by the upward shift of the labor demand curve.

Under full information ($\psi_\pi = 0$), the labor supply curve remains at the initial position, so the increase in TFP pushes the economy to a new equilibrium ***B*** featuring higher output $y' > y$ and higher real wages.

If households beliefs about inflation are anchored ($\psi_\pi > 0$), the reduction in prices perceived by households is lower in magnitude than the reduction in prices by firms. As a result, they perceive a more moderate increase in real wages and consume less in response. The information friction creates a wedge in labor demand that shifts the labor supply, offsetting part of the increase in output due to the rise in productivity. The equilibrium with incomplete information, indicated by point ***C***, features higher real wages but an output level that lies between the initial output level y and the full information level y'.

The second diagram of Figure 2 shows the effect of a technology shock on the aggregate demand and supply of this economy. As indicated in the previous paragraph, the productivity shock moves the equilibrium from point **A** to point **C**.

When the information wedge in aggregate demand is present ($\chi > 0$), there is a second round of attenuation: The slope of aggregate demand is now steeper, the increase in output is further mitigated, and the technology shock is largely deflationary. This additional attenuation comes from the fact that households under-react to the increase in permanent income from the technology shock, as they don't perceive completely the fall in the aggregate price level that follows the shock. The final equilibrium, indicated by point D, features a more modest increase in output than what would be obtained under full information.

5.2 Learning by Shopping as Rational Inattention to Aggregate Prices

Section 4 discussed the channels through which learning by shopping affects business cycles and their relationship with the monetary policy stance. An underlying assumption in this analysis was that the informativeness of households shopping experiences, as measured by σ_ϵ^2, was constant and exogenously given.

I now relax this assumption and allow the households to choose the precision of these signals. Following the Rational Inattention literature pioneered by Sims (2003), I allow households to choose the attention allocated to inflation by trading the costs of ignoring inflation with the costs of acquiring information about this variable.

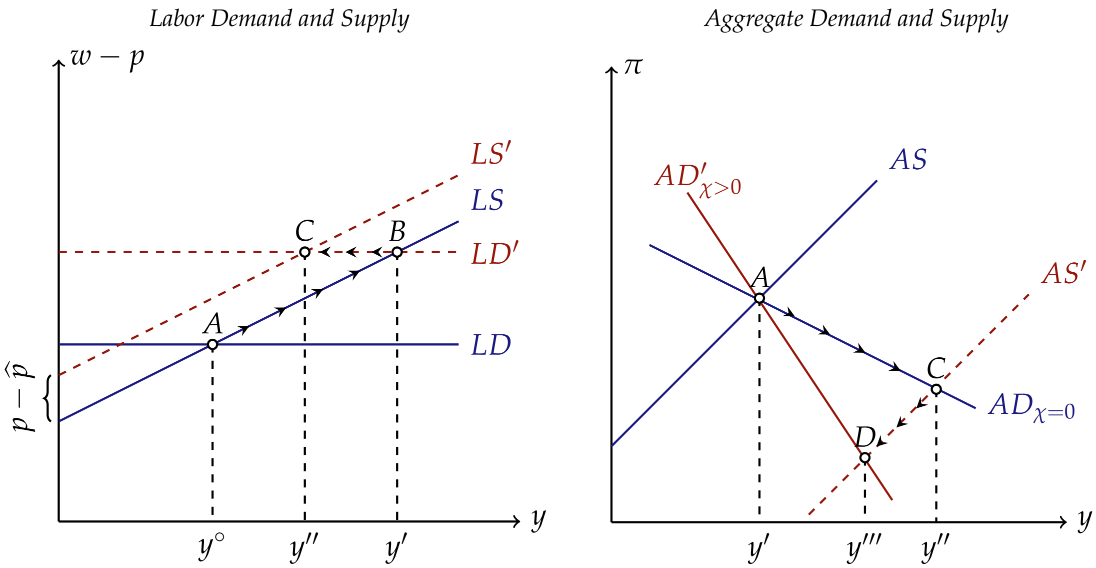


Figure 2: Learning by shopping and the attenuation of technology shocks

Notes: The figure illustrates how learning by shopping attenuates the impact of a positive productivity shock. The first panel shows the the reduced impact of the shock in the labor supply due to the differences in the perceived real wage between households and firms. The second panel shows the further attenuation arising from households' under-reaction to the increase in permanent income produced by the shock.

The costs of ignoring inflation. Inattention to aggregate inflation results in consumption, savings and labor supply decisions that differ from those that the household would take under full information. It follows that an agent that ignores inflation achieves a lower welfare (1), compared to a fully attentive agent.

To derive an expression for the welfare costs incurred by household i from ignoring aggregate inflation, I replace the budget constraint (5) in the objective function (1). A log-quadratic approximation approximation of the household's objective function around the non-stochastic steady state yields the following result.

Proposition 8. *(**The costs of inattention to inflation**) The welfare cost for household i from having incomplete information about aggregate inflation is given by:*

$$\mathcal{IC}_\pi = -\frac{1}{2} C^{1-\sigma} \mathrm{E}_{-1} \sum_{t=0}^{\infty} \beta^t \left\{ \sigma \left(c_{i,t} - c_{i,t}^* \right)^2 + \mathcal{M}^{-1} \varphi \left(n_{i,t} - n_{i,t}^* \right)^2 \right\}, \tag{45}$$

where $c_{i,t} - c_{i,t}^*$ and $n_{i,t} - n_{i,t}^*$ are the deviations of the household consumption and labor supply from their full-information counterparts. These deviations are given by

$$c_{i,t} - c_{i,t}^* = -\frac{1}{\sigma} \beta \left\{ \nu_{i,t+1}^\pi + \sum_{k=1}^{\infty} \beta^k \left\{ \nu_{i,t+k+1}^\pi - \phi_\pi \nu_{i,t+k}^\pi \right\} \right\} + \beta \chi \sum_{k=0}^{\infty} \beta^k \nu_{i,t+k}, \tag{46}$$

$$n_{i,t} - n_{i,t}^* = \frac{1}{\varphi} v_{i,t} - \frac{\sigma}{\varphi} \left(c_{i,t} - c_{i,t}^* \right). \tag{47}$$

Proof. See Appendix A.8. □

The above proposition shows how the private costs of ignoring inflation are proportional to the magnitude of their misperception, which in turn produce sub-optimal consumption and labor supply decisions. The deviations (46) and (47) closely resemble the information wedges affecting the aggregate Euler equation (20) and the aggregate labor supply (20). This is not a coincidence, as these wedges are the result of this deviations at the micro level.

Importantly, Proposition (45) shows that these deviations have second-order effects on the welfare of each household. Nevertheless, they can have first-order effects on the behavior of macroeconomic variables, as illustrated in the previous section. As observed by Akerlof and Yellen (1985), this is also the case with menu costs models. The two frictions represent forms of near-rationality where individual agents face second-order losses from deviating of the frictionless behavior, but their deviations can give rise to comovement between output and inflation.

The costs of acquiring information. In absence of any constraint on information acquisition, households would choose to observe inflation with infinite precision. Following the Rational Inattention literature, I assume that the utility costs of acquiring information are a linear in Shannon's mutual information function.

Formally, let $\boldsymbol{p}^T \equiv \{p_t\}_{t=0}^T$ and $\boldsymbol{s}_i^T \equiv \{s_{i,t}\}_{t=0}^T$ denote the history of the aggregate price and the signals received by household i up to period T. Let $H\left(\boldsymbol{p}^T\right)$ and $H\left(\boldsymbol{p}^T | \boldsymbol{s}_i^T\right)$ denote the entropy and conditional entropy of $\boldsymbol{p}^T$ and $\boldsymbol{s}_i^T$. I assume that the agent's flow cost of information at time t is given by $\omega \mathbb{I}\left(\boldsymbol{p}^T, \boldsymbol{s}^T\right)$, where

$$\mathbb{I}\left(\boldsymbol{p}^T, \boldsymbol{s}_i^T\right) \equiv H\left(\boldsymbol{p}^T\right) - H\left(\boldsymbol{p}^T | \boldsymbol{s}_i^T\right), \tag{48}$$

is the mutual information between of $\boldsymbol{p}^T$ and $\boldsymbol{s}^T$, and $\omega > 0$ is the marginal cost of a unit of information.[26] Intuitively, mutual information measures the reduction in uncertainty about aggregate prices $\boldsymbol{p}^T$ from observing $\boldsymbol{s}^T$. The cost $\omega > 0$ can be interpreted as an opportunity cost, measured in utility terms, of devoting attention to tracking inflation.

The attention problem of the household. We are now in position to write the attention problem of the household. In period $t = -1$, before choosing consumption, each household chooses the precision of the signals that it receives in the following periods. In each

[26] See Cover and Thomas (2012) for a comprehensive introduction to information theory.

period $t \geq 0$, the expectation of current and future prices is formed given the sequence of all signals that the household has received up to that point in time.

Formally, the problem of the household is to choose σ_ϵ^2 to maximize

$$-\frac{1}{2}C^{1-\sigma}\mathrm{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left\{\sigma\left(c_{i,t}-c_{i,t}^*\right)^2+\mathcal{M}^{-1}\varphi\left(n_{i,t}-n_{i,t}^*\right)^2-\omega\mathbb{I}\left(\boldsymbol{p}^T,\boldsymbol{s}^T\right)\right\} \tag{49}$$

subject to the signal structure (6) and equations (46) and (47) defining $c_{i,t}-c_{i,t}^*$ and $n_{i,t}-n_{i,t}^*$.

Solving this type of problem is only possible using numerical methods. But we can use the same assumptions used in Section 3 to get some intuition on the form of the optimal attention to inflation.

To do so, recall that, conditional on a value of σ_ϵ^2, the equilibrium inflation is given by equation (37). Since both inflation and signals are Normal random variables that follow i.i.d. processes, mutual information (48) takes a simple form:[27]

$$\mathbb{I}\left(\pi_t,\pi_{i,t}^*\right)=\frac{1}{2}\log\left(1+\frac{\mathrm{Var}\left[\pi_t\right]}{\sigma_\epsilon^2}\right).$$

We can see that mutual information is increasing in the signal-to-noise ratio of the signals. Notice also that the agent is atomistic and takes the variance of inflation $\mathrm{Var}\left[\pi_t\right]$ as given. The fact that inflation follows an i.i.d. process implies that the deviations (46) and (47) simplify to

$$c_{i,t}-c_{i,t}^*=\beta\chi\nu_{i,t},$$
$$n_{i,t}-n_{i,t}^*=\frac{1}{\varphi}\left(1-\sigma\beta\chi\right)\nu_{i,t}$$

We can thus rewrite the information acquisition problem (49) as

$$\min_{\sigma_\epsilon^2}\quad\Omega\mathrm{E}_{i,-1}\left[\nu_{i,t}^2\right]+\omega\log\left(1+\frac{\mathrm{Var}\left[\pi_t\right]}{\sigma_\epsilon^2}\right) \tag{50}$$

where the parameter Ω is given by

$$\Omega\equiv C^{1-\sigma}\left(\sigma\left(\beta\chi\right)^2+\mathcal{M}^{-1}\frac{1}{\varphi}\left(1-\sigma\beta\chi\right)^2\right). \tag{51}$$

This parameter summarizes the costs from sub-optimal attention to inflation. Finally, we can use the well-known regression lemma for the distribution of bivariate normal variables to get:

$$\mathrm{E}_{i,-1}\left[\nu_{i,t}^2\right]=\mathrm{Var}_{i,t}\left[\pi_t|\pi_{i,t}^*\right]=\mathrm{Var}\left[\pi_t\right]\left(1-\frac{1}{\mathrm{Var}\left[\pi_t\right]+\sigma_\epsilon^2}\right)$$

[27] Here I use the natural logarithm to express information units in nats, as opposed to bits, in which case, the logarithm has base 2.

We can thus take first order conditions of (50) and solve for σ_ϵ^2 to arrive to the following result.

Proposition 9. ***(Rational attention to inflation)*** *The degree of anchoring ψ_π^* of a rationally inattentive household is given by*

$$\psi_\pi^* = \max\left\{\min\left\{\frac{\omega}{\Omega}, 1\right\}, 0\right\}.$$

Proof. See Appendix A.9. □

Proposition 9 shows that the optimal level of inattention ψ_π^* is common across households, increasing in the costs of acquiring information ω, and decreasing in the utility costs of ignoring inflation Ω. These costs, as defined in (51), reflect the suboptimal choice of consumption and labor that arises from misperceiving the inflation rate.

Perhaps surprisingly, the inflation rate or its variance does not show up in the optimal level of anchoring. The optimal choice of attention σ_ϵ^2 in this simple setting requires households to keep a constant signal-to-noise ratio $\text{Var}\left[\pi_t\right]/\sigma_\epsilon^2$, so the precision of their signals σ_ϵ^2 moves one-to-one with the variance of inflation to keep the signal-to-noise ratio constant.

The constancy of ψ_π^* is a consequence of the simplifying assumptions used to derive this expression. In the general model, where both shocks and information evolve slowly over time, this will not be the case.

6 Learning by Shopping: A Quantitative Analysis

In this section, I explore the robustness of the theoretical results derived in the previous sections in the general setting where shocks are persistent and information about the aggregate price level evolves slowly over time.

6.1 Quantitative Model

The quantitative model used in this section introduces three modifications over the model presented in Section 3. in three directions. First, I allow shocks to aggregate demand and TFP to be persistent over time. Specifically, I assume that $z_{i,t}$ is given by

$$z_{i,t} = \rho_{AD} z_{i,t} + \eta_t^{AD} + \xi_{i,t}^z,$$
$$\eta_t^{AD} \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_{AD}^2\right), \quad \xi_{i,t}^z \overset{iid}{\sim} \mathcal{N}\left(0, \zeta_x^2\right),$$

and that aggregate productivity a_t follows an AR(1) process of the form

$$a_t = \rho_{AS} a_{t-1} + \eta_t^{AS}; \quad \eta_t^{AS} \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_{AS}^2\right).$$

Second, I assume that the total expenditures $M_{i,t}$ are now subject to an auxiliary noise shock that prevents households from infering the past price level at the beggining of each period.[28] For this reason, the prior belief of the household no longer coincides with the past aggregate price level, and beliefs are updated slowly over time.

Finally, I introduce the following assumption on the initial information set of households.

Assumption 2. *The initial information set, $\mathcal{I}_{i,-1}$, contains an infinite history of signals.*

The above assumption is common in the rational inattention literature.[29] It will allow me to abstract from purely deterministic transitional dynamics in the conditional second moments of beliefs. This guarantees that the Kalman gain coefficients characterizing the learning process of households are constant over time. Nevertheless, the Kalman gains will still be endogenous objects determined in equilibrium.

6.2 Calibration

Most of the parameters in the model can be calibrated using values commonly found in the business cycle literature. The only non-standard parameter is given by the cost of acquiring information ω, which plays a crucial role in the model.

The anchor in the data. There is not direct way to measure ω in the data. However, there is a direct relationship between ω and the Kalman gain coefficients κ_h in (23). Since these parameters are directly related to households' beliefs, we can use data on households' inflation expectations to estimate some of these κ_h indirectly.

To do so, notice we can use (25) to write the average belief across households about inflation in the past year as

$$\widehat{\pi}_t^{YoY} = (1-\psi_\pi)\,\pi_t^{YoY} + \psi_\pi \widehat{\pi}_{t|t-1}^{YoY} + u_t, \tag{52}$$

where $\psi_\pi \equiv 1-(\kappa_0-\kappa_{12})$ is the *anchor of year-on-year inflation* (assuming a monthly frequency) and $u_t \equiv (1-\psi_\pi)\int_0^1 \left\{s_{i,t-12} - \widehat{p}_{i,t-12|t-1}\right\} di$ is a proportional to the signals acquired by the household in the previous year.

[28]This shock plays a similar role to the auxiliary shocks introduced in Section **??**. One can think about M_t as the credit card bill, and interpret these shocks as unexpected fees and charges in the credit card bill that prevent each household from infering the aggregate price level P_t from just looking at it's credit card bill.

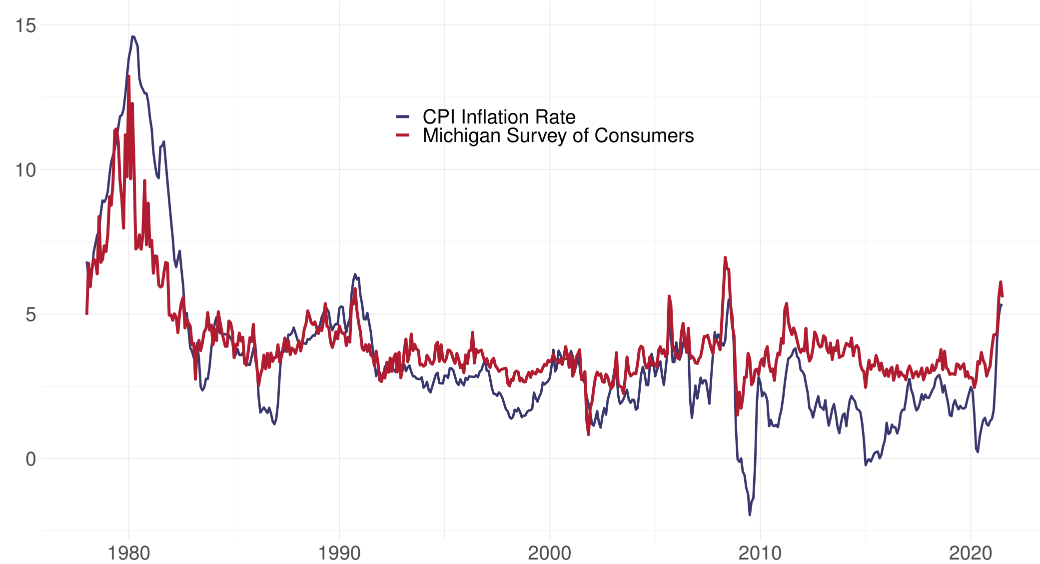
[29]See, for instance, Woodford (2009), Mackowiak and Wiederholt (2009), and Maćkowiak and Wiederholt (2015). The assumption also provides a useful benchmark to compare the model with models where firms are inattentive. Recently, Afrouzi and Yang (2021) developed computational methods to solve rational inattention models without this assumption. The effects of relaxing this assumption are not trivial. Afrouzi and Yang (2021) find large effects in the study of firms' pricing decisions, while Mackowiak and Wiederholt (2020) find it little effect for the propagation of news shocks.

Estimating this specification requires data on both inflation perceptions and inflation expectations.

Unfortunately, such a dataset is not available for the U.S. However, recent evidence suggests that the inflation expectations reported by participants of the Michigan Survey are very similar to their perceptions about current inflation. Using special questionaries introduced in this survey, Axelrod et al. (2018) find that one third of respondents report the same perception of inflation as their reported expectation, and one sixth reports expectations that deviate from their perception by less than a one percentage point. Similar evidence is provided by Jonung (1981) for a cross-section of swedish households, Armantier et al. (2016) for a cross-section of households in the NY FED Survey of Consumers Expectations and Coibion et al. (2018a), Candia et al. (2021) for firms in New Zeland and the U.S. . This evidence suggests that the measures of expectations available from the Michigan Survey are good proxies of households perceptions about inflation.

Under this interpretation, we can estimate (52) directly from the data shown in Figure 3, by replacing $\widehat{\pi}^{YoY}_{t|t-1}$. by $\widehat{\pi}^{YoY}_{t-1}$.

Figure 3: Inflation Expectations by Households in the U.S.



Notes: The figure shows the average expectation about future inflation held by participants of the *Survey of Consumers* conducted by the University of Michigan. The red line shows the average belief about how prices will change in the following 12 months. The blue line shows the 12-month CPI inflation rate provided by the U.S. Bureau of Labor Statistics.

Table 1 shows the results of this exercise using monthly data and the year-on-year CPI inflation rate as as a proxy for π_t^{YoY} in (52).[30]

[30] A similar specification was used by Carroll (2003) to estimate the relationship between households' expectations and those of professional forecasters.

The estimated specifications 1 and 2 shows that the specification offers a good approximation of the data. Moreover, when the constant term is dropped, we cannot reject the null that the sum of the coefficients associated to π_t^{YoY} and $\widehat{\pi}_{t|t-1}^{YoY}$ is equal to one, as one would expect when one of the variables is a distributed lag of the other.

Specifications 3-6 show that the value of this coefficient has not been stable over time. In the period preceding Volcker's tenure as Fed Chairman, the anchoring coefficient was almost half the size of the coefficient in the period post-Volcker.[31] This is consistent with the prediction of the model that the degree of anchoring is endogenous and depends on the conduct of monetary policy.

Table 1: Inattention to Inflation in the Michigan Survey of Consumers

Estimating Equation: $\widehat{\pi}_t^{YoY} = \beta_0 + \beta_1 \pi_t^{YoY} + \beta_2 \widehat{\pi}_{t-1}^{YoY} + \epsilon_t$						
Equation	$\widehat{\beta}_0$	$\widehat{\beta}_1$	$\widehat{\beta}_2$	$\widehat{\psi}_\pi$	**Sample**	R^2
1	0.315	0.106	0.804		1978M01 - 2019M12	0.94
	(0.092)	(0.022)	(0.046)			
2		0.047	0.941	0.833		0.98
		(0.015)	(0.016)	(0.042)		
3	0.495	0.166	0.721		1978M01 - 1982M12	0.83
	(0.538)	(0.052)	(0.099)			
4		0.166	0.775	0.466		0.98
		(0.051)	(0.080)	(0.145)		
5	0.689	0.086	0.695		1983M01 - 2019M12	0.70
	(0.118)	(0.018)	(0.046)			
6		0.052	0.946	0.847		0.98
		(0.016)	(0.016)	(0.044)		

Notes: This table shows the estimated value of. $\widehat{\pi}_t^{YoY}$ is the period-t mean of the Michigan survey measure of households expectations over the next 12 months. π_t^{YoY} is the CPI inflation rate between period t and $t-12$. Standard deviation of the corresponding estimate is shown in parentheses. Standard errors are corrected for heteroskedasticity and autocorrelation following a Newey-West ((1987)) procedure with twelve lags.. The corresponding level of anchoring at quarterly frequency is computed as $\widehat{\psi}_\pi = \widehat{\beta}_2^3$ and its standard erros are calculated using the Delta method.

Parameter values. The baseline calibration of the model is summarized in Table 2 . I assume each period is a quarter and set the discount factor β to 0.99, so that the steady-state real risk-free rate is 4 percent. I set the inverse of elasticity of intertemporal substitution σ to 2, consistent with the baseline estimates by Crump et al. (2015).[32] I set the inverse of the Frisch elasticity of labor supply φ to 4, following Chetty et al. (2011). I set the elasticity of substitution across varieties to $\varepsilon = 6$, the Calvo index of price rigidities θ to 0.75 (consistent

[31] The results are similar if we instead split the sample in 1990m01, which is a break commonly used in the literature estimating the slope of the Phillips curve.

[32] Notice that these authors estimate an Euler equation by individual that corresponds directly to equation 15 in this model.

with an average price duration of one year), and the inflation coefficient in the Taylor rule ϕ_π to 1.5.

I fix $\rho_{TFP} = \rho_{AD} = \rho$ to make sure results are not driven by differences in persistence. I then calibrate simultaneously parameters $\left(\rho, \sigma^2_{AD}, \sigma^2_{TFP}, \omega\right)$ to match four moments of the data for the post-Volcker period: 1) The correlation and variance of quarterly Core CPI inflation observed, 2) the share of variance in output explained by non-technology shocks estimated in Galí and Gambetti (2009), and 3) a value of ψ_π of 0.85, in line with the estimated values of specification 6 in Table 1.

The value of ω necessary to match the desired calibration implies that the costs of acquiring information $\omega\mathbb{I}\left(\cdot\right)$ are equivalent to 0.2% of the steady-state level of consumption of each household. These costs are small, in line with the predictions from Proposition (8).

Table 2: Model Calibration

Parameter	**Value**	**Description**	**Source / Target**
Assigned			
β	0.99	Discount factor	quarterly frequency
σ	2	Inv. elasticity of intertemporal subs.	Crump et al. (2015)
φ	4	Inv. Frisch elasticity of labor supply	Chetty et al. (2011)
ε	6	Elasticity of substitution	avg. price markup of 20%
θ	0.75	1 - Prob. of adjusting prices	avg. price duration of 4 quarters
ϕ_π	1.5	Interest rate rule coefficient	Taylor (1993)
Calibrated			
ρ	0.93	Persistence of shocks	$\text{Corr}\left[\pi_t, \pi_{t-1}\right] = 0.79$
σ_{TFP}	0.85×10^{-3}	Std. Dev. TFP shock	$\text{SD}\left[y_t \mid z_t\right] / \text{SD}\left[y_t\right] = 0.70$
σ_{AD}	3.81×10^{-3}	Std. Dev. AD shock	$\text{SD}\left[\pi_t, \pi_{t-1}\right] = 0.79$
ω	1.35×10^{-3}	Information cost	$\psi_\pi = 0.85$

Notes: The table presents the baseline parameters for the quantitative model. The first panel shows the value of the parameters assigned based on values commonly found in the literature. The second panel shows the value of four parameters calibrated jointly to match different moments in the data.

6.3 The amplification of nominal rigidities

Aggregate demand shock. I start by studying the dynamic response of the variables in the model to an expansionary shock in aggregate demand z_t. Figure 4 shows the response to a one standard deviation shock under three different scenarios.

The first scenario, in blue, shows the response when price stickiness is the only friction present ($\omega = 0$, $\theta > 0$). The shock produces comovement between output, inflation and employment, as is the usual case in with this nominal rigidity. Notice that in this case, the inflation perceived by households is identical to the actual inflation rate.

The second scenario, in red, shows the response when learning by shopping is the only friction present in the model ($\omega > 0$, $\theta = 0$). Consistent with the analysis in the previous section, this information friction also produces comovement between inflation and output. In contrast to the scenario with price stickiness, the dynamics of output and employment show additional persistence and a hump-shaped response to the shock. The additional persistence comes from the slow response of inflation perceptions, which now under-react to true inflation in the first periods.

The third scenario, in yellow, shows the response when both sticky prices and learning by shopping are present. The interaction of the two frictions amplifies the response of output and employment dramatically: The response on impact is 2.5 times larger than the combined response of the two frictions considered in isolation. The interaction also adds additional persistence to the response of output and makes inflation less volatile compared to the case where only the information friction is present.

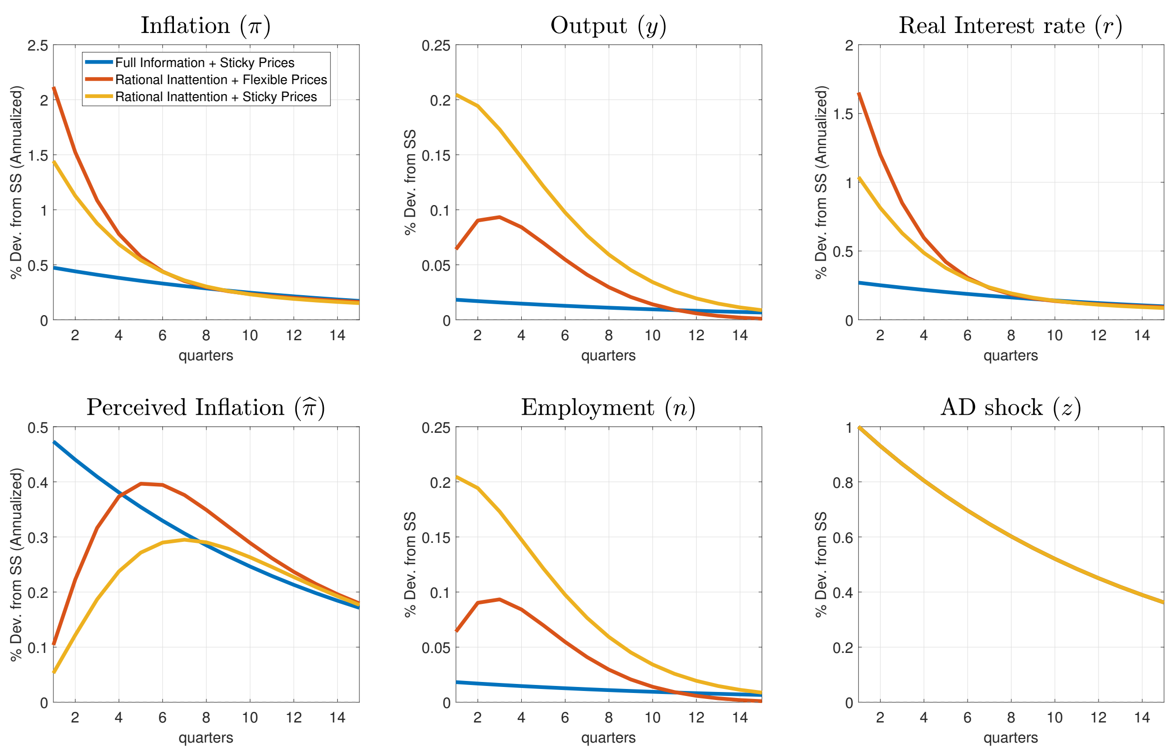


Figure 4: Dynamic Responses to an Aggregate Demand Shock

Notes: The figure plots the impulse responses to a one standard deviation expansionary shock in aggregate demand under the baseline calibration. The blue line shows the response when price stickiness is the only friction present. The red line shows the corresponding response when *learning by shopping* is the only friction present. The yellow line shows the response when both frictions are present.

Technology shock. Figure 5 shows the response of a one standard deviation positive shock to TFP under the three different scenarios considered before. We can see that the presence of learning by shopping attenuates the response of output, but the attenuation is much larger when this information friction is interacted with price stickiness. The figure shows how this mitigation comes from the lower perceived inflation, which makes households perceive a real wage that is lower than the actual wage. The effect of this inattention is observed in the amplification of the negative response of employment to this shock.

The results suggest that, when households have incomplete information about inflation, technology shocks are even less likely to generate positive comovement between employment and output.

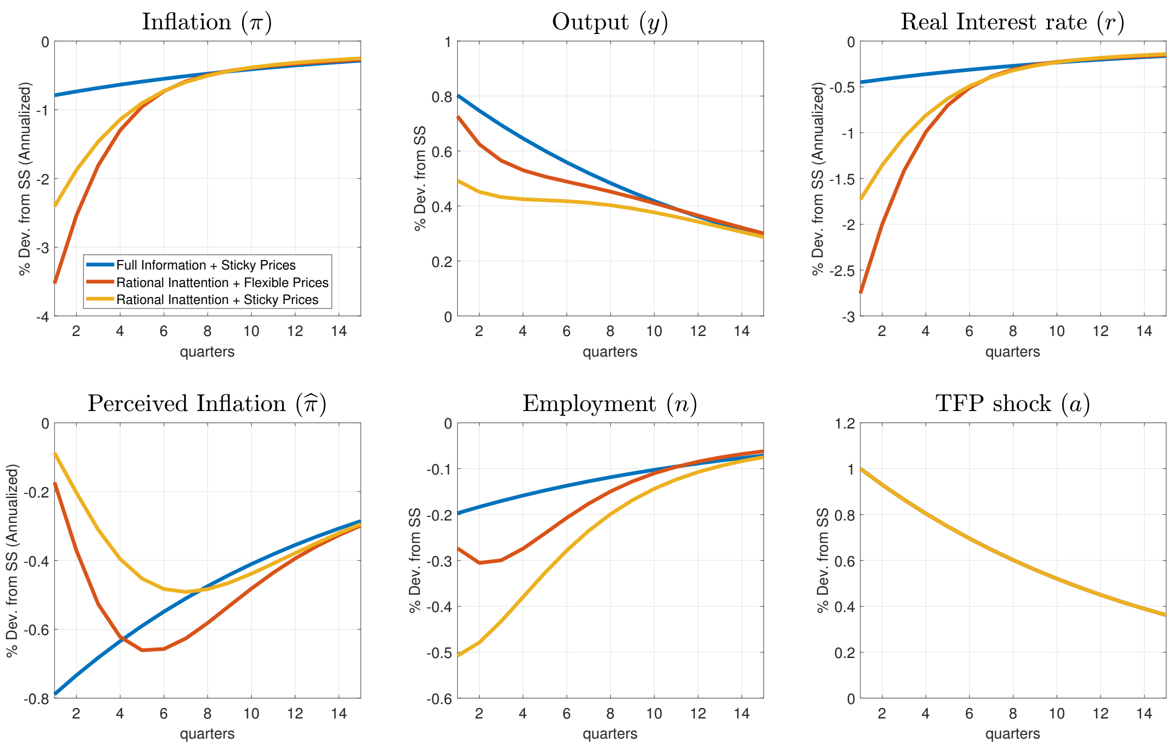


Figure 5: Dynamic Responses to a Technology Shock

Notes: The figure plots the impulse responses to a one standard deviation expansionary shock in TFP under the baseline calibration. The blue line shows the response when price stickiness is the only friction present. The red line shows the corresponding response when *learning by shopping* is the only friction present. The yellow line shows the response when both frictions are present.

6.4 The effect of a change in the monetary policy stance

I now use the calibrated model to analyze the impact of a change in monetary policy to more dovish stance. Following Maćkowiak and Wiederholt (2015) and Afrouzi and Yang (2021), I lower the coefficient of the monetary policy rule from $\phi_\pi = 1.5$ to $\phi_\pi = 1$, and compare the moments and the IRFs implied by the model with those observed in the pre-Volcker period. This will allow us to test some of the theoretical predictions from the last sections and see if they can match the U.S. experience from the last decades.

Table 3 shows the results of this exercise, and Figures 6 and 7 show the IRFs in response to an aggregate demand and technology shock in each scenario.

Table 3: Moments Implied by the Model Under Different Calibrations

	Full Sample	Pre-Volcker ($\phi_\pi = 1$)		Post-Volcker ($\phi_\pi = 1.5$)			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
		Endog.		Full.	Exog.	Endog.	
Moment	Data	Info.	Data	Info.	Info.	Info.	Data
$SD(\pi_t)$	0.65	0.83	0.88	0.12	0.18	0.24	0.24
$Corr(\pi_t, \pi_{t-1})$	0.87	0.93	0.85	0.92	0.77	0.79	0.79
ψ_π	0.83	0.23	0.46	0	0.57	0.85	0.85
$SD[y_t\|z_t] / SD[y_t]$	-	0.62	0.76	0.09	0.49	0.70	0.70
$SD[y_t\|z_t]^{Post} / SD[y_t\|z_t]^{Pre}$			-	0.15	0.81	1.23	0.59
$SD[y_t\|a_t]^{Post} / SD[y_t\|a_t]^{Pre}$			-	1.27	1.14	0.98	0.83

Notes: The table presents moments of the data and simulated series from the model under four counterfactual scenarios. Column (1) displays the moments of the data for the full sample. Column (2) and (3) show the moments implied by a more dovish monetary policy and compares them with the moments in the data for the Pre-Volcker era. Column (4) shows the corresponding moments when housheolds have full information about inflation. Column (5) shows the moments implied by the model when information is exogenos and fixed to its value in the Pre-volcker era. Column (6) shows the moments implied by the baselina calibration, and Column (7) shows the corresponding moments for the Post-Volcker era.

Column (2) shows that the calibrated model predicts an increase in volatility and persistence of inflation after such a policy change. This prediction is consistent with the higher volatility and persistence in Core CPI inflation observed during the pre-Volcker era, as shown in column (3). Such a policy leads to an unanchoring of households' inflation perceptions, but its magnitude is larger than what is suggested by the estimates from Table 1.

Column (2) shows that the model also predicts that the share of volatility of GDP explained by aggregate demand shocks decreases under a more dovish policy. This is a result of the amplification of demand shocks produced by inattention to inflation.[33]

To gain further insight on the impact of having incomplete information in the model, column (4) shows the corresponding moments when only nominal rigidities are present in the model. The results show that a model without information frictions has a hard time rationalizing the fall in persistence of inflation observed after an increase in ϕ_π. It also predicts a strong reduction in the volatility of inflation and the contribution of demand shocks that goes beyond what is observed in the data.

[33]This observation may seem at odds with the evidence of a lower contribution of demand shocks to fluctuations in output by Galí and Gambetti (2009). One possible explanation is that the change in policy considered here is larger than the change that actually took place during these periods, as shown by the "overshooting" of the inflation anchor, and that the sample used for this calibration includes an additional decade of observations after the Great Recession.

To highlight the importance of taking into account the endogenous response of households to changes in policy, consider an scenario where the value of σ_ϵ^2 is fixed to the value implied by the counterfactual exercise of column (2). We can interpret this scenario as an experiment where a policy maker in the pre-Volcker era tries to predict the effects of an increase in ϕ_π.

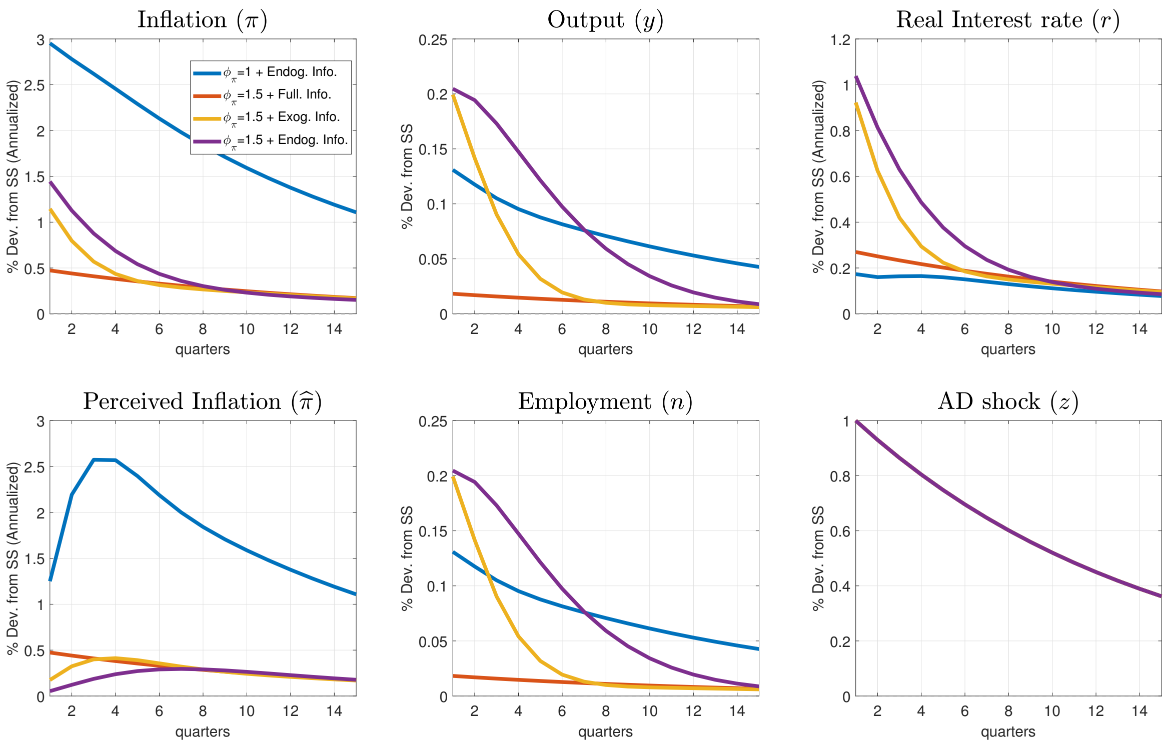


Figure 6: Dynamic Response to an Aggregate Demand Shock under Different Policy Scenarios

Notes: The figure plots the impulse responses to a one standard deviation expansionary shock in aggregate demand under different scenarios. The blue line shows the response when $\phi_\pi = 1.0$ and all other parameters remain as in the baseline calibration. The red line shows the response when $\phi_\pi = 1.5$ and price stickiness is the only friction present. The yellow line shows the response when $\phi_\pi = 1.5$ and both price stickiness and *learning by shopping* are present, but information is exogenous, and attention is fixed to its value of the first scenario. The purple line shows the corresponding response when $\phi_\pi = 1.5$, both frictions are present, and agents choose the attention to inflation, making information endogenous.

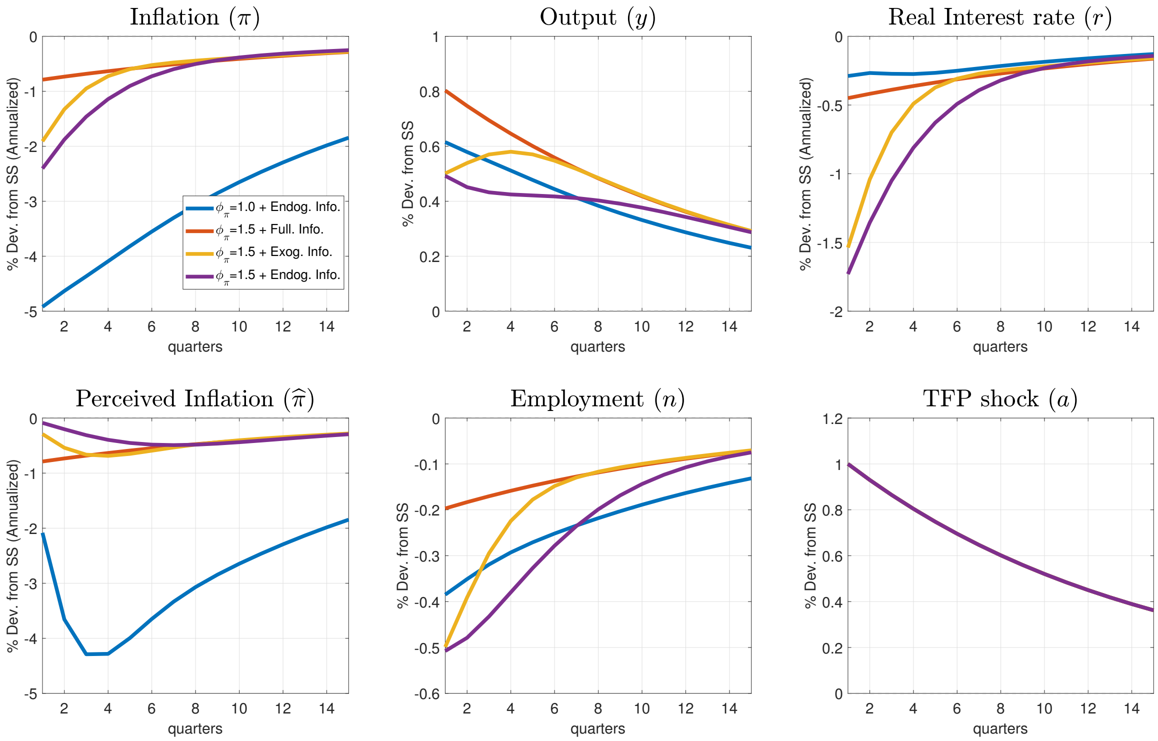


Figure 7: Dynamic Response to a Technology Shock under Different Policy Scenarios

Notes: The figure plots the impulse responses to a one standard deviation expansionary shock in TFP under different scenarios. The blue line shows the response when $\phi_\pi = 1.0$ and all other parameters remain as in the baseline calibration. The red line shows the response when $\phi_\pi = 1.5$ and price stickiness is the only friction present. The yellow line shows the response when $\phi_\pi = 1.5$ and both price stickiness and *learning by shopping* are present, but information is exogenous, and attention is fixed to its value of the first scenario. The purple line shows the corresponding response when $\phi_\pi = 1.5$, both frictions are present, and agents choose the attention to inflation, making information endogenous.

Column (5) shows that the policy maker using a model with exogenous information would correctly predict the fall in the volatility and persistence of inflation, as well as part of the anchoring of inflation beliefs. Moreover, the exercise would predict that the change in policy would result in a response of output to demand shocks that is larger but short-lived, as shown in Figure 6.

But this model would give an incomplete picture of the effects of the policy. The success in reducing the volatility of inflation lowers the incentives to learn about inflation. Households rationally choose to ignore inflation even more, producing further re-anchoring of their beliefs. As shown in Figure 6, this re-anchoring of beliefs amplifies the persistence in output from demand shocks and mitigates even further the impact of technology shocks.

This exercise suggests that inattention to inflation is actually a sign of success by the central bank on its mission of stabilizing inflation. It also suggests that this success has unintended consequences: In a new environment with less volatile inflation, the informational content of aggregate prices is reduced. As a result, technology shocks become inflationary and demand shocks become the principal driver of business cycles.

7 Concluding Remarks

Since Lucas (1973), a large part of the literature has viewed information frictions as a substitute to menu costs. In this paper, I challenged that view by showing that both types of frictions can coexist, and their interaction gives rise to business cycles that are dominated by exogenous shifts in aggregate demand.

To conclude, let me suggest future research avenues. The results of this paper offer a nuanced view of the role of the transmission mechanisms of monetary policy. It suggests that the success in stabilizing inflation has also altered the transmission of technology and non-technology shocks into the economy. What is the optimal monetary policy should in this environment is an open question. Should central banks target some measure of households beliefs? Can policies that aim to inform the general audience about news on the inflation rate backfire?

A second interesting avenue of future work is to study the impact of oil shocks when households have incomplete information about aggregate inflation. As argued forcefully by Coibion and Gorodnichenko (2015), energy prices are the main driver of fluctuations in households' inflation expectations in the short term. Can these shocks produce exogenous differences in households and firms' perceptions that feedback in the inflation rate? The framework presented in this paper provides a starting point to answer this question.

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A Proofs

A.1 Proof of Proposition 1

I derive the Aggregate Euler Equation 20 when both aggregate demand and supply shocks are present. To do so, I start by deriving an expression characterizing individual consumption as a beauty contest from the households' budget constraint and first order conditions. I then characterize the aggregate demand of this economy as a function of the information wedges defined in the proposition. I conclude by characterizing the information wedges as a function of misperception of the price level and the inflation rate[34].

Consumption as a beauty contest

Define $r^Z_{i,t+1} \equiv r_{i,t+1} + z_{i,t+1} - z_{i,t}$, $r_{i,t+1} \equiv i_{i,t} - \pi_{t+1}$, $\widehat{p}_{i,t|s} \equiv \mathrm{E}_{i,s} p_t$ and $\widehat{\pi}_{i,t|s} \equiv \mathrm{E}_{i,s}\pi_t$. The labor supply and Euler equation of household i (15) and (14) can be expressed as:

$$w_{i,t} - \widehat{p}_{i,t|t} = \sigma c_{i,t} + \varphi n_{i,t}, \tag{A.53}$$

$$c_{i,t} = \mathrm{E}_{i,t} c_{i,t+1} - \frac{1}{\sigma}\mathrm{E}_{i,t} r^Z_{i,t+1}. \tag{A.54}$$

The aggregate labor supply can be expressed as

$$w_t - \widehat{p}_{t|t} = (\varphi + \sigma)\, c_t - \varphi a_t, \tag{A.55}$$

where $\widehat{p}_{t|t} \equiv \int_0^1 \widehat{p}_{i,t|t} di$. Log-linearizing the end-of period budget constraint (5) gives

$$c_{i,t} + b^R_{i,t} = \beta^{-1} b^R_{i,t-1} + \omega_W \left(w^R_{i,t} + n_{i,t} \right) + \omega_D d^R_{i,t}, \tag{A.56}$$

where the superscript R denotes the variable deflated by the price level p_t. The constants $\omega_W = \frac{WN}{PC}$ and $\omega_D \equiv \frac{D}{PC}$ denote steady-state ratios[35]. Using A.53, and recalling that $\nu_{i,t|t} \equiv p_t - \widehat{p}_{i,t|t}$, we can rewrite (A.56) as

$$\left(1 + \frac{\sigma}{\varphi}\omega_W\right) c_{i,t} + b^R_{i,t} = \beta^{-1} b^R_{i,t-1} + e^R_{i,t},$$

with

$$e^R_{i,t} \equiv \omega_W \left(1 + \frac{1}{\varphi}\right) w^R_{i,t} + \frac{\omega_W}{\varphi}\nu_{i,t} + \omega_D d^R_{i,t}. \tag{A.57}$$

[34] All variables in lower case denote log-deviations from steady-state, except for the price level $p_t \equiv \log P_t$ and the bond holdings, which are written as $b_{i,t} = B_{i,t}/C$ and $b^R_{i,t} = B_{i,t}/\left(P_t C\right)$, where C denotes the steady-state level of consumption. This redefinition takes care of the issue that $B = 0$ in the non-stochastic steady-state, and is standard in the literature (see, for instance, Woodford (2011), Angeletos and Lian (2018), and Angeletos and Lian (2021)).

[35] Notice that $\omega_W = \mathcal{M}^{-1}$, and $\omega_W + \omega_D = 1$.

Solving for $b^R_{i,t-1}$, iterating forward and using the transversality condition and taking expectations yields

$$b^R_{i,t-1} + \sum_{k=0}^{\infty} \beta^k \mathrm{E}_{i,t} e^R_{i,t+k} = \left(\frac{\varphi + \sigma\omega_W}{\varphi}\right) \sum_{k=0}^{\infty} \beta^k \mathrm{E}_{i,t} c_{i,t+k}. \tag{A.58}$$

The next step is to use the Euler equation of the household to rewrite (A.58). Iterating (A.54) forward and using the fact that the law of iterated expectations holds, conditional on the household information set, we have

$$c_{i,t} = -\frac{1}{\sigma} \sum_{h=0}^{\infty} \mathrm{E}_{i,t} r^Z_{i,t+h+1}. \tag{A.59}$$

Multiplying this equation by β^k in different periods and adding the respective equations yields:

$$\sum_{k=0}^{\infty} \beta^k E_{i,t} c_{i,t+k} = -\frac{1}{\sigma} \mathrm{E}_{i,t} \left[\sum_{k=0}^{\infty} \sum_{h=0}^{\infty} \beta^k r^Z_{i,t+h+k+1} \right]. \tag{A.60}$$

Now, notice that

$$\begin{aligned}
\sum_{k=0}^{\infty} \sum_{h=0}^{\infty} \beta^k r^Z_{i,t+h+k+1} &= \left(\frac{1-\beta}{1-\beta}\right) r^Z_{i,t+1} + \frac{1-\beta^2}{1-\beta} r^Z_{i,t+2} + \frac{1-\beta^3}{1-\beta} r^Z_{i,t+3} + \\
&= \sum_{k=0}^{\infty} \left(\frac{1-\beta^{k+1}}{1-\beta}\right) r^Z_{i,t+k+1} \\
&= \frac{1}{1-\beta} \left(\sum_{k=0}^{\infty} r^Z_{i,t+k+1} - \beta \sum_{k=0}^{\infty} \beta^k r^Z_{i,t+k+1} \right).
\end{aligned}$$

We can use the previous expression back in (A.60) and use (A.59) to get

$$\begin{aligned}
\sum_{h=0}^{\infty} \beta^h E_{i,t} c_{i,t+h} &= -\frac{1}{\sigma} \left(\frac{1}{1-\beta}\right) \left(\sum_{k=0}^{\infty} \mathrm{E}_{i,t} r^Z_{i,t+k+1} - \beta \sum_{k=0}^{\infty} \beta^k \mathrm{E}_{i,t} r^Z_{i,t+k+1} \right) \\
&= \left(\frac{1}{1-\beta}\right) \left(\left\{ -\frac{1}{\sigma} \sum_{k=0}^{\infty} \mathrm{E}_{i,t} r^Z_{i,t+k+1} \right\} + \frac{1}{\sigma} \beta \sum_{k=0}^{\infty} \beta^k \mathrm{E}_{i,t} r^Z_{i,t+k+1} \right) \\
&= \left(\frac{1}{1-\beta}\right) \left(c_{i,t} + \frac{1}{\sigma} \beta \sum_{k=0}^{\infty} \beta^k \mathrm{E}_{i,t} r^Z_{i,t+k+1} \right).
\end{aligned} \tag{A.61}$$

We can plug back this expression in (A.58) and solve for $c_{i,t}$ to get:

$$c_{i,t} = -\frac{1}{\sigma} \beta \left\{ \sum_{k=0}^{\infty} \beta^k \mathrm{E}_{i,t} r^Z_{i,t+k+1} \right\} + (1-\beta) \left(\frac{\varphi}{\varphi + \sigma\omega_W}\right) \sum_{k=0}^{\infty} \beta^k \mathrm{E}_{i,t} e^R_{i,t+k} + (1-\beta) \left(\frac{\varphi}{\varphi + \sigma\omega_W}\right) b^R_{i,t-1} \tag{A.62}$$

Integrating this expression across households and using the market clearing condition for bonds yields:

$$c_t = -\frac{1}{\sigma}\beta \int_0^1 \left[\sum_{k=0}^{\infty} \beta^k \mathrm{E}_{i,t} r^Z_{i,t+k+1}\right] di + (1-\beta)\left(\frac{\varphi}{\varphi+\sigma\omega_W}\right)\int_0^1 \left[\sum_{k=0}^{\infty} \beta^k \mathrm{E}_{i,t} e^R_{i,t+k}\right] di. \quad \text{(A.63)}$$

The next step is to express the second term in brackets as a function of aggregate consumption. To do so, start by observing that, up to a first-order approximation, the (real) dividends of each firm are given by

$$d^R_{j,t} = y_{j,t} + \left(\frac{1}{1-\omega_W}\right) p^R_{j,t} - \left(\frac{\omega_W}{1-\omega_W}\right)\left(w^R_{j,t} - a_t\right).$$

Integrating across firms and using the market clearing condition in goods market yields

$$d^R_{i,t} \equiv \int d^R_{i,j,t} dj = c_t - \left(\frac{\omega_W}{1-\omega_W}\right)\left(w^R_t - a_t\right).$$

Replacing this expression in (A.57), we have

$$\begin{aligned} e^R_{i,t+k} =& \omega_W \left(1+\frac{1}{\varphi}\right) w^R_{i,t+k} + \frac{\omega_W}{\varphi} \nu_{i,t+k|t+k} + (1-\omega_W)\, d^R_{i,t+k} \\ =& \frac{\omega_W}{\varphi}\left(w^R_{i,t+k} + \nu_{i,t|t}\right) + (1-\omega_W)\, c_{t+k} + \omega_W \left(w^R_{i,t+k} - w^R_{t+k}\right) + \omega_W a_{t+k} \\ =& \frac{\omega_W}{\varphi}\left(w_{i,t+k} - \widehat{p}_{i,t+k|t+k}\right) + (1-\omega_W)\, c_{t+k} + \omega_W \left(w^R_{i,t+k} - w^R_{t+k}\right) + \omega_W a_{t+k}. \end{aligned} \quad \text{(A.64)}$$

Households understand that their differences in nominal wages and dividends are unpredictable. They hold rational expectations and can use (A.64) and the aggregate labor supply (A.53) to get:

$$\begin{aligned} \mathrm{E}_{i,t} e^R_{i,t+k} &= \frac{\omega_W}{\varphi} \mathrm{E}_{i,t}\left[w_{t+k} - \widehat{p}_{t+k|t+k}\right] + (1-\omega_W)\, \mathrm{E}_{i,t} c_{t+k} + \omega_W \mathrm{E}_{i,t} a_{t+k} \\ &= \frac{\omega_W}{\varphi} \mathrm{E}_{i,t}\left[(\varphi+\sigma)\, c_{t+k} - \varphi a_{t+k}\right] + (1-\omega_W)\, \mathrm{E}_{i,t} c_{t+k} + \omega_W \mathrm{E}_{i,t} a_{t+k} \\ &= \left(\frac{\omega_W}{\varphi}(\varphi+\sigma) + (1-\omega_W)\right) \mathrm{E}_{i,t}\left[c_{t+k}\right] - \omega_W \mathrm{E}_{i,t} a_{t+k} + \omega_W \mathrm{E}_{i,t} a_{t+k} \\ &= \left(\frac{\varphi+\sigma\omega_W}{\varphi}\right) \mathrm{E}_{i,t} c_{t+k}. \end{aligned} \quad \text{(A.65)}$$

Plugging these results back in (A.63), we get

$$c_t = -\frac{1}{\sigma}\beta \sum_{k=0}^{\infty} \beta^k \int_0^1 \mathrm{E}_{i,t} r^Z_{i,t+k+1} di + (1-\beta)\left[\sum_{k=0}^{\infty} \beta^k \int_0^1 \mathrm{E}_{i,t} c_{t+k} di\right]. \tag{A.66}$$

This equation characterizes aggregate consumption as a beauty contest, in the spirit of Angeletos and Lian (2018).

Aggregate demand as a function of information wedges

Start by writing the first term in (A.66) as

$$\begin{aligned}
\sum_{k=0}^{\infty} \beta^k \int_0^1 \mathrm{E}_{i,t} r^Z_{i,t+k+1} di &= \sum_{k=0}^{\infty} \beta^k \int_0^1 \mathrm{E}_{i,t} \left\{ r_{i,t+k+1} + z_{i,t+k+1} - z_{i,t+k} \right\} di \\
&= \sum_{k=0}^{\infty} \beta^k \int_0^1 \mathrm{E}_{i,t} r_{i,t+k+1} di + \sum_{k=0}^{\infty} \beta^k \int_0^1 \mathrm{E}_{i,t} \left\{ z_{i,t+k+1} - z_{i,t+k} \right\} di \\
&= \sum_{k=0}^{\infty} \beta^k \left\{ \int_0^1 \mathrm{E}_{i,t} r_{i,t+k+1} - \mathbb{E}_t r_{t+k+1} \right\} di + \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t r^Z_{t+k+1},
\end{aligned}$$

where $\mathbb{E}_t$ is the full information operator. Now, rewrite the second term in (A.66) as

$$\begin{aligned}
\sum_{k=1}^{\infty} \beta^k \int_0^1 \mathrm{E}_{i,t} c_{t+k} di &= \sum_{k=1}^{\infty} \beta^k \int_0^1 \mathrm{E}_{i,t} c_{t+k} di - \sum_{k=1}^{\infty} \beta^k \mathbb{E}_t c_{t+k} + \sum_{k=1}^{\infty} \beta^k \mathbb{E}_t c_{t+k} \\
&= \sum_{k=1}^{\infty} \beta^k \int_0^1 \left\{ \mathrm{E}_{i,t} c_{t+k} - \mathbb{E}_t c_{t+k} \right\} di + \sum_{k=1}^{\infty} \beta^k \mathbb{E}_t c_{t+k}.
\end{aligned}$$

We can now rewrite the equation (A.66) as:

$$c_t = -\frac{1}{\sigma}\beta \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t r^Z_{t+k+1} + (1-\beta) \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t c_{t+k} + \beta \mathcal{X}_t, \tag{A.67}$$

where

$$\begin{aligned}
\mathcal{X}_t &\equiv \mathcal{H}_t + \mathcal{R}_t, \\
\mathcal{H}_t &\equiv \left(\frac{1-\beta}{\beta}\right) \sum_{k=0}^{\infty} \beta^k \int_0^1 \left\{ \mathrm{E}_{i,t} c_{t+k} - \mathbb{E}_t c_{t+k} \right\} di, \\
\mathcal{R}_t &\equiv -\frac{1}{\sigma} \sum_{k=0}^{\infty} \beta^k \int_0^1 \left\{ \mathrm{E}_{i,t} r_{i,t+k} - \mathbb{E}_t r_{t+k} \right\} di.
\end{aligned}$$

All that is left is to write this expression in recursive form. To do so, start by taking out c_t

from the RHS of (A.67) and solve c_t to get

$$c_t = -\frac{1}{\sigma}\sum_{k=1}^{\infty}\beta^{k-1}\mathbb{E}_t r^Z_{t+k} + \left(\frac{1-\beta}{\beta}\right)\sum_{k=1}^{\infty}\beta^k\mathbb{E}_t c_{t+k} + \mathcal{X}_t. \tag{A.68}$$

Writing this equation in $t+1$ and taking expectations in t yields

$$\mathbb{E}_t c_{t+1} = -\frac{1}{\sigma}\sum_{k=1}^{\infty}\beta^{k-1}\mathbb{E}_t r^Z_{t+k+1} + \left(\frac{1-\beta}{\beta}\right)\sum_{k=1}^{\infty}\beta^k\mathbb{E}_t c_{t+k+1} + \mathbb{E}_t\mathcal{X}_{t+1}.$$

Using this expression back in (A.68), we get

$$\begin{aligned}
c_t &= -\frac{1}{\sigma}\mathbb{E}_t r^Z_{t+1} + (1-\beta)\,\mathbb{E}_t c_{t+1} + \left\{-\frac{1}{\sigma}\sum_{k=2}^{\infty}\beta^{k-1}\mathbb{E}_t r^Z_{t+k} + \left(\frac{1-\beta}{\beta}\right)\sum_{k=2}^{\infty}\beta^k\mathbb{E}_t c_{t+k}\right\} + \mathcal{X}_t \\
&= -\frac{1}{\sigma}\mathbb{E}_t r^Z_{t+1} + (1-\beta)\,\mathbb{E}_t c_{t+1} + \beta\left\{-\frac{1}{\sigma}\sum_{k=1}^{\infty}\beta^{k-1}\mathbb{E}_t r^Z_{t+k+1} + \left(\frac{1-\beta}{\beta}\right)\sum_{k=1}^{\infty}\beta^k\mathbb{E}_t c_{t+k+1}\right\} + \mathcal{X}_t \\
&= -\frac{1}{\sigma}\mathbb{E}_t r^Z_{t+1} + (1-\beta)\,\mathbb{E}_t c_{t+1} + \beta\left\{\mathbb{E}_t c_{t+1} - \mathbb{E}_t\mathcal{X}_{t+1}\right\} + \mathcal{X}_t \\
&= -\frac{1}{\sigma}\mathbb{E}_t r^Z_{t+1} + \mathbb{E}_t c_{t+1} + \mathcal{X}_t - \beta\mathbb{E}_t\mathcal{X}_{t+1}.
\end{aligned}$$

Finally, replacing c_t by y_t using market clearing, we get

$$y_t = -\frac{1}{\sigma}\mathbb{E}_t r^Z_{t+1} + \mathbb{E}_t y_{t+1} + \mathcal{X}_t - \beta\mathbb{E}_t\mathcal{X}_{t+1}. \tag{A.69}$$

Information wedges as a function of price perceptions

Start by considering first the term $\mathcal{H}_t$. Using (A.65), we have

$$\mathrm{E}_{i,t}c_t = \left(\frac{\varphi}{\varphi+\sigma\omega_W}\right)\mathrm{E}_{i,t}e^R_{i,t+k}.$$

The previous expression implies:

$$\int_0^1\left\{\mathrm{E}_{i,t}e^R_{i,t+k} - \mathbb{E}_t e^R_{t+k}\right\}di = \int_0^1\left\{\mathrm{E}_{i,t}e_{i,t+k} - \mathbb{E}_t e_{t+k}\right\}di + \nu_{t+k|t} = \nu_{t+k|t}. \tag{A.70}$$

The second equality is a consequence of the observation that households hold rational expectations and observe in t everything in their nominal income $e_{i,t}$. It follows that:

$$\int_0^1\left\{\mathrm{E}_{i,t}c_{t+k} - \mathbb{E}_t c_{t+k}\right\}di = \left(\frac{\varphi}{\varphi+\sigma\omega_W}\right)\nu_{t+k|t}.$$

Using this result, we can express the information wedge $\mathcal{H}_t$ as:

$$\mathcal{H}_t \equiv \left(\frac{1-\beta}{\beta}\right)\sum_{k=0}^{\infty}\beta^k\int_0^1\left\{\mathrm{E}_{i,t}c_{t+k}-\mathbb{E}_t c_{t+k}\right\}di = \underbrace{\left(\frac{1-\beta}{\beta}\right)\left(\frac{\varphi}{\varphi+\sigma\omega_W}\right)}_{\chi}\sum_{k=0}^{\infty}\beta^k v_{t+k|t},$$

where

$$\chi \equiv \left(\frac{1-\beta}{\beta}\right)\left(\frac{\varphi}{\varphi+\sigma\omega_W}\right) = \left(\frac{1-\beta}{\beta}\right)\left(\frac{\mathcal{M}\varphi}{\mathcal{M}\varphi+\sigma}\right).$$

Next, consider the the wedge on the real interest rate $\mathcal{R}_t$. We have

$$\begin{aligned}\int_0^1\left\{\mathrm{E}_{i,t}r_{i,t+k+1}-\mathbb{E}_t r_{t+k+1}\right\}di &= \int_0^1\left\{\mathrm{E}_{i,t}\left\{i_{i,t+k}-\pi_{t+k+1}\right\}-\mathbb{E}_t\left\{i_{t+k}-\pi_{t+k+1}\right\}\right\}di\\ &= \int_0^1 \mathrm{E}_{i,t}i_{i,t+k}di - \int_0^1 \mathrm{E}_{i,t}\pi_{t+k+1}di - \mathbb{E}_t i_{t+k} + \mathbb{E}_t\pi_{t+k+1}.\end{aligned}$$

For $k=0$, households know their own interest rate. Therefore

$$\int_0^1\left\{\mathrm{E}_{i,t}r_{i,t}-\mathbb{E}_t r_{t+k+1}\right\}di = \int_0^1\left\{-\widehat{\pi}_{t+1|t}+\mathbb{E}_t\pi_{t+1}\right\} = v^{\pi}_{t+1|t}.$$

For $k>0$, we can use the monetary policy rule, which is common knowledge across households, to get

$$\begin{aligned}\int_0^1\left\{\mathrm{E}_{i,t}r_{i,t+k+1}-\mathbb{E}_t r_{t+k+1}\right\}di &= \left\{\phi_\pi\left(\widehat{\pi}_{t+k|t}-\pi_{t+k}\right)-\left(\widehat{\pi}_{t+k+1|t}-\mathbb{E}_t\pi_{t+k+1}\right)\right\}\\ &= \left(-\phi_\pi v^{\pi}_{t+k|t}+v^{\pi}_{t+k+1|t}\right). \qquad \text{(A.71)}\end{aligned}$$

Using this result, we can express the information wedge $\mathcal{R}_t$ as:

$$\mathcal{R}_t \equiv -\frac{1}{\sigma}\sum_{k=0}^{\infty}\beta^k\int_0^1\left\{\mathrm{E}_{i,t}r_{i,t+k}-\mathbb{E}_t r_{t+k}\right\}di = -\frac{1}{\sigma}\left(v^{\pi}_{t+1|t}+\sum_{k=1}^{\infty}\beta^k\left\{v^{\pi}_{t+k+1|t}-\phi_\pi v^{\pi}_{t+k|t}\right\}\right).$$

This concludes the poor of the proposition.

A.2 Proof of Proposition 2

Using the definition of Δ_π and the values of α_{AD} and α_{PC} in 32 and 34 yields:

$$\Delta_\pi \equiv \frac{\sigma+\varphi}{\lambda+(\sigma+\varphi)\,\sigma^{-1}\phi_\pi+(1-(\sigma+\varphi)\,\chi)\,\psi_\pi} > 0.$$

The definition of ψ_π in equation (29) and equation the law of motion of inflation 36 imply:

$$1-\psi_\pi = \frac{\mathrm{Var}\left[\pi_t\right]}{\mathrm{Var}\left[\pi_t\right]+\sigma^2_\epsilon} = \frac{\Delta^2_\pi\sigma^2_u}{\Delta^2_\pi\sigma^2_u+\sigma^2_\epsilon},$$

where $\sigma^2_u = \mathrm{Var}\left[u_t\right]$ and $u_t \equiv \widetilde{z}_t - \widetilde{a}_t$. Defining $q \equiv \sigma^2_u/\sigma^2_\epsilon$ as the signal-to-noise ration in households signals, we can rewrite the previous expression as

$$1-\psi_\pi=\Delta_\pi^2 q\psi_\pi.$$

Now, notice that the LHS of this equation is decreasing in ψ_π, is equal to 1 when $\psi_\pi=0$ and equal to 0 when $\psi_\pi=1$. The RHS is equal to 0 when $\psi_\pi=0$ and equal to some positive constant when $\psi_\pi=1$. Continuity of the RHS guarantees the existence of a solution of this equation. To prove its uniqueness, it is sufficient to show that the RHS is always increasing in ψ_π. Start by observing that

$$\begin{aligned}\frac{\partial\Delta_\pi}{\partial\psi_\pi}&\equiv-\left(\frac{\sigma+\varphi}{\left(\lambda+(\sigma+\varphi)\,\sigma^{-1}\phi_\pi+(1-(\sigma+\varphi)\,\chi)\,\psi_\pi\right)^2}\right)(1-(\sigma+\varphi)\,\chi)\\&=-\left(\frac{1-(\sigma+\varphi)\,\chi}{\lambda+(\sigma+\varphi)\,\sigma^{-1}\phi_\pi+(1-(\sigma+\varphi)\,\chi)\,\psi_\pi}\right)\Delta_\pi.\end{aligned}$$

It follows that

$$\begin{aligned}\frac{\partial RHS}{\partial\psi_\pi}&=q\left(\Delta_\pi^2+2\Delta_\pi\frac{\partial\Delta_\pi}{\partial\psi_\pi}\psi_\pi\right)\\&=q\Delta_\pi^2\left(1-2\psi_\pi\left(\frac{1-(\sigma+\varphi)\,\chi}{\lambda+(\sigma+\varphi)\,\sigma^{-1}\phi_\pi+(1-(\sigma+\varphi)\,\chi)\,\psi_\pi}\right)\right)\\&=q\Delta_\pi^2\left(\frac{\lambda+(\sigma+\varphi)\,\sigma^{-1}\phi_\pi-(1-(\sigma+\varphi)\,\chi)\,\psi_\pi}{\lambda+(\sigma+\varphi)\,\sigma^{-1}\phi_\pi+(1-(\sigma+\varphi)\,\chi)\,\psi_\pi}\right).\end{aligned}$$

The condition that $(\sigma+\varphi)\,\chi\in(0,1)$ guarantees this derivative is always positive.

A.3 Proof of Proposition 3

From Proposition 2, we can define the equilibrium level of anchoring implicitly as the root of the following equation:

$$F(\lambda,\psi_\pi(\lambda))=q\Delta_\pi^2\psi_\pi+\psi_\pi-1,$$

with

$$\Delta_\pi\equiv\frac{\sigma+\varphi}{\lambda+(\sigma+\varphi)\,\sigma^{-1}\phi_\pi+(1-(\sigma+\varphi)\,\chi)\,\psi_\pi}.$$

Taking the partial derivatives of $F(\cdot)$, we get:

$$\partial F/\partial\psi=\Delta_\pi^2 q\left\{1-\frac{2\,(1-(\sigma+\varphi)\,\chi)}{(\sigma+\varphi)^2}\Delta_\pi\psi_\pi\right\}>0.$$

$$\partial F/\partial\lambda=-2q\frac{1}{(\sigma+\varphi)^2}\Delta_\pi^3\psi_\pi,<0$$

$$\partial F/\partial\phi_\pi = -2q\frac{1}{(\sigma+\varphi)^2}\Delta_\pi^3\psi_\pi(\sigma+\varphi)\sigma^{-1} < 0,$$

From the Implicit Function Theorem, it follows that:

$$\frac{\partial\psi}{\partial\lambda} = \frac{2\Delta_\pi\psi_\pi}{(\sigma+\varphi)^2 - 2(1-(\sigma+\varphi)\chi)\Delta_\pi\psi_\pi} > 0$$

$$\frac{\partial\psi}{\partial\phi_\pi} = \frac{2\Delta_\pi(\sigma+\varphi)\sigma^{-1}\psi_\pi}{(\sigma+\varphi)^2 - 2(1-(\sigma+\varphi)\chi)\Delta_\pi\psi_\pi} > 0.$$

This concludes the proof of the proposition.

A.4 Proof of Proposition 4

Follows directly from (36).

A.5 Proof of Proposition 5

Let $\Phi \equiv (\sigma+\varphi)\sigma^{-1}\phi_\pi$ and $X \equiv (1-(\sigma+\varphi)\chi)$. Equation (41) implies that the response considered in each scenario is given by

$$\left[\frac{\partial y_t}{\partial z_t}\right]^{LS} = \sigma^{-1}\left(\frac{\psi^{LS}}{\Phi + X\psi^{LS}}\right),$$

$$\left[\frac{\partial y_t}{\partial z_t}\right]^{SP} = \sigma^{-1}\left(\frac{\lambda}{\lambda+\Phi}\right),$$

$$\left[\frac{\partial y_t}{\partial z_t}\right]^{LS+SP} = \sigma^{-1}\left(\frac{\lambda+\psi^{LS+SP}}{\lambda+\Phi+X\psi^{LS+SP}}\right),$$

where ψ_π^{LS} and ψ_π^{LS+SP} is the degree of anchoring in the corresponding scenario. Amplification is obtained when

$$\mathcal{A} \equiv \sigma^{-1}\left(\left\{\underbrace{\frac{\psi_\pi^{LS+SP}}{\lambda+\Phi+X\psi_\pi^{LS+SP}} - \frac{\psi_\pi^{LS}}{\Phi+X\psi_\pi^{LS}}}_{B}\right\} - \left\{\underbrace{\frac{\lambda}{\lambda+\Phi} - \frac{\lambda}{\lambda+\Phi+X\psi_\pi^{LS+SP}}}_{C}\right\}\right) > 0.$$

Notice that C is allways positive. It follows that amplification is only possible if B is also positive. For this to be the case, we must have

$$\frac{\psi_\pi^{LS+SP}}{\psi_\pi^{LS}} > 1 + \frac{\lambda}{\Phi}.$$

Now, B can be simplified

$$
\begin{aligned}
B \equiv \frac{\psi_\pi^{LS+SP}}{\lambda+\Phi+X\psi_\pi^{LS+SP}} - \frac{\psi_\pi^{LS}}{\Phi+X\psi_\pi^{LS}} &= \frac{\psi_\pi^{LS+SP}\left(\Phi+X\psi_\pi^{LS}\right)-\psi_\pi^{LS}\left(\lambda+\Phi+X\psi_\pi^{LS+SP}\right)}{\left(\lambda+\Phi+X\psi_\pi^{LS+SP}\right)\left(\Phi+X\psi_\pi^{LS}\right)} \\
&= \frac{\psi_\pi^{LS+SP}\left(\Phi+X\psi_\pi^{LS}\right)-\psi_\pi^{LS}\left(\lambda+\Phi+X\psi_\pi^{LS+SP}\right)}{\left(\lambda+\Phi+X\psi_\pi^{LS+SP}\right)\left(\Phi+X\psi_\pi^{LS}\right)} \\
&= \frac{\psi_\pi^{LS+SP}\Phi-\psi_\pi^{LS}\left(\lambda+\Phi\right)}{\left(\lambda+\Phi+X\psi_\pi^{LS+SP}\right)\left(\Phi+X\psi_\pi^{LS}\right)}.
\end{aligned}
$$

And C can be simplified as

$$
\begin{aligned}
C \equiv \frac{\lambda}{\lambda+\Phi} - \frac{\lambda}{\lambda+\Phi+X\psi_\pi^{LS+SP}} &= \lambda\left\{\frac{\lambda+\Phi+X\psi_\pi^{LS+SP}-\lambda-\Phi}{\left(\lambda+\Phi\right)\left(\lambda+\Phi+X\psi_\pi^{LS+SP}\right)}\right\} \\
&= \psi_\pi^{LS+SP}\left(\frac{\lambda X}{\left(\lambda+\Phi\right)\left(\lambda+\Phi+X\psi_\pi^{LS+SP}\right)}\right)
\end{aligned}
$$

We can thus rewrite $\mathcal{A}$ as

$$
\begin{aligned}
\mathcal{A} &= \sigma^{-1}\left(\psi_\pi^{LS+SP}\left(\frac{\lambda X}{\left(\lambda+\Phi\right)\left(\lambda+\Phi+X\psi_\pi^{LS+SP}\right)}\right)-\frac{\psi_\pi^{LS+SP}\Phi-\psi_\pi^{LS}\left(\lambda+\Phi\right)}{\left(\lambda+\Phi+X\psi_\pi^{LS+SP}\right)\left(\Phi+X\psi_\pi^{LS}\right)}\right) \\
&= \frac{\sigma^{-1}\psi_\pi^{LS}}{\left(\lambda+\Phi+X\psi_\pi^{LS+SP}\right)}\left\{\frac{\psi_\pi^{LS+SP}}{\psi_\pi^{LS}}\left[\frac{\lambda X}{\lambda+\Phi}-\frac{\Phi}{\Phi+X\psi_\pi^{LS}}\right]+\frac{\lambda+\Phi}{\Phi+X\psi_\pi^{LS}}\right\}
\end{aligned}
$$

It follows that a sufficient condition for $\mathcal{A}>0$ is

$$
\frac{\psi_\pi^{LS+SP}}{\psi_\pi^{LS}}\left[\frac{\lambda X}{\lambda+\Phi}-\frac{\Phi}{\Phi+X\psi_\pi^{LS}}\right]+\frac{\lambda+\Phi}{\Phi+X\psi_\pi^{LS}}>0.
$$

We can rewrite this as

$$
\frac{\psi_\pi^{LS+SP}}{\psi_\pi^{LS}}\left(\frac{\lambda X}{\lambda+\Phi}\right)+\frac{\lambda+\Phi}{\Phi+X\psi_\pi^{LS}}>\left(\frac{\psi_\pi^{LS+SP}}{\psi_\pi^{LS}}\right)\frac{\Phi}{\Phi+X\psi_\pi^{LS}}.
$$

The necessary condition for amplification implies that the previous equation holds whenever

$$
\frac{\psi_\pi^{LS+SP}}{\psi_\pi^{LS}}\left(\frac{\lambda X}{\lambda+\Phi}\right)+\frac{\lambda+\Phi}{\Phi+X\psi_\pi^{LS}}>\left(1+\frac{\lambda}{\Phi}\right)\frac{\Phi}{\Phi+X\psi_\pi^{LS}},
$$

or, equivalently

$$
\frac{\psi_\pi^{LS+SP}}{\psi_\pi^{LS}}\left(\frac{\lambda X}{\lambda+\Phi}\right)>0.
$$

The assumption that $(\sigma+\varphi)\chi<1$ implies that $X>0.$, and Proposition 3 implies that

$\psi_\pi^{LS+SP} > \psi_\pi^{LS}$. It follows that $\mathcal{A} > 0$.

A.6 Proof of Proposition 6

The result follows from the definition of α_{AS}^* and Proposition A.3.

A.7 Proof of Proposition 7

Follows directly from (36).

A.8 Proof of Proposition 8

Start by replacing (11) in the definition of total expenditures in the budget constraint (5). Using this expression, we can express the consumption level of each household as:

$$C_{i,t} = R_{i,t} B_{i,t-1}^R + W_{i,t}^R N_{i,t} + D_{i,t}^R - B_{i,t}^R,$$

where $B_{i,t}^R \equiv B_{i,t}/P_t$, $W_{i,t}^R \equiv W_{i,t}/P_t$, and $D_{i,t}^R \equiv D_{i,t}/P_t$, and $R_t = Q_{i,t}^{-1} P_{t-1}/P_t$. denotes the real interest rate. Substituting this expression in 2, we can express the period utility of the household as:

$$Z_{i,t} \left\{ \frac{1}{1-\sigma} \left(R_{i,t} B_{i,t-1}^R + W_{i,t}^R N_{i,t} + D_{i,t}^R - B_{i,t}^R \right)^{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} - \frac{1}{1-\sigma} \right\}.$$

Rewrite the expression in brackets as:

$$\frac{1}{1-\sigma} C^{1-\sigma} \left(\beta^{-1} e^{r_{i,t}} b_{i,t-1}^R + \omega_W e^{w_{i,t}^R + n_{i,t}} + \omega_D e^{d_{i,t}^R} - b_{i,t}^R \right)^{1-\sigma} - N^{1+\varphi} \frac{e^{(1+\varphi) n_{i,t}}}{1+\varphi} - \frac{1}{1-\sigma},$$

where the notation is the same used in the proof of in the proof of Proposition (1).[36] Multiplying this expression by β^t, summing over all $t = 0, 1, \ldots$ and taking expectation conditional on information in $t = -1$, we can rewrite the objective (1) as:

$$\begin{aligned} \mathcal{W}\left(\boldsymbol{x}_{i,t}; \boldsymbol{y}_{i,t} \right) = \mathrm{E}_{i,-1} \sum_{t=0}^{\infty} \beta^t Z_{i,t} \Bigg\{ & \frac{1}{1-\sigma} C^{1-\sigma} \left(\beta^{-1} e^{r_{i,t}} b_{i,t-1}^R + \omega_W e^{w_{i,t}^R + n_{i,t}} + \omega_D e^{d_{i,t}^R} - b_{i,t}^R \right)^{1-\sigma} \\ & - \omega_W C^{1-\sigma} \frac{e^{(1+\varphi) n_{i,t}}}{1+\varphi} - \frac{1}{1-\sigma} \Bigg\} \end{aligned}$$

where $\boldsymbol{x}_{i,t} \equiv \left(b_{i,t}^R, n_{i,t} \right)'$ is a vectors of choice variables, and $\boldsymbol{y}_{i,t} \equiv \left(r_{i,t-1}, w_{i,t}^R, d_{i,t}^R, z_{i,t} \right)$, is a vector of variables and prices taken as given by the household.

[36] Notice that the labor supply equation (12) implies that $N^{1+\varphi} = \omega_W C^{1-\sigma}$.

Now, let $\boldsymbol{x}_{i,t}^*$ denote the optimal action of household i under full information and assume for simplicity that $b_{i,-1}^* = b_{-1}$. Under some regularity conditions that guarantee that $\boldsymbol{x}_{i,t} - \boldsymbol{x}_{i,t}^*$ has finite second moments,[37] we can take a quadratic approximation of $\mathcal{W}(\cdot)$ around the origin to derive the following expression of the expected loss in utility for any action $\boldsymbol{x}_{i,t} \neq \boldsymbol{x}_{i,t}^*$:

$$
\begin{aligned}
\mathcal{IC}_\pi(\boldsymbol{x}_{i,t}) &\equiv \mathcal{W}\left(\boldsymbol{x}_{i,t}; \boldsymbol{y}_{i,t}\right) - \mathcal{W}\left(\boldsymbol{x}_{i,t}^*; \boldsymbol{y}_{i,t}\right) \qquad \text{(A.72)} \\
&\approx \mathrm{E}_{-1} \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2} \left(\boldsymbol{x}_{i,t} - \boldsymbol{x}_{i,t}^*\right)^T \boldsymbol{\mathcal{H}}_0 \left(\boldsymbol{x}_{i,t} - \boldsymbol{x}_{i,t}^*\right) + \left(\boldsymbol{x}_t - \boldsymbol{x}_{i,t}^*\right)^T \boldsymbol{\mathcal{H}}_1 \left(\boldsymbol{x}_{i,t+1} - \boldsymbol{x}_{i,t+1}^*\right) \right\} + t.i.p.
\end{aligned}
$$

where the matrices of derivatives $\boldsymbol{\mathcal{H}}_0$ and $\boldsymbol{\mathcal{H}}_1$ are given by:

$$
\boldsymbol{\mathcal{H}}_0 = -C^{1-\sigma} \begin{bmatrix} \sigma\left(1+\beta^{-1}\right) & -\sigma\omega_W \\ -\sigma\omega_W & \omega_W\left(\varphi + \sigma\omega_W\right) \end{bmatrix},
$$

$$
\boldsymbol{\mathcal{H}}_1 = C^{1-\sigma} \begin{bmatrix} \sigma & -\sigma\omega_W \\ 0 & 0 \end{bmatrix}.
$$

At this stage, we can follow the same steps in Proposition 2 of the Online Appendix of Maćkowiak and Wiederholt (2015) to rewrite $\mathcal{IC}_\pi$ as a function of $\widetilde{c}_{i,t} \equiv c_{i,t} - c_{i,t}^*$ and $\widetilde{n}_{i,t} \equiv n_{i,t} - n_{i,t}^*$. First, note that (A.56) implies the optimal actions $\boldsymbol{x}_t^*$ under full information satisfy

$$
c_{i,t}^* = \beta^{-1} b_{i,t-1}^* - b_{i,t}^* + \omega_W \left(n_{i,t}^* + w_{i,t}^R\right) + \omega_D d_{i,t}^R.
$$

Consequently, we can express bond holdings deviations $\widetilde{b}_{i,t} = b_{i,t} - b_{i,t}^*$ as

$$
\widetilde{b}_{i,t} = \beta^{-1}\widetilde{b}_{i,t-1} + \omega_W \widetilde{n}_{i,t} - \widetilde{c}_{i,t}
$$

Iterating this expression backwards, we can rewrite it recursively as

$$
\widetilde{b}_{i,t} = \Delta_{i,t}^N - \Delta_{i,t}^C
$$

with $\Delta_{i,t}^C = \widetilde{c}_{i,t} + \beta^{-1}\Delta_{i,t-1}^C$, $\Delta_{i,t}^N = \omega_W \widetilde{n}_{i,t} + \beta^{-1}\Delta_{i,t-1}^N$ and $\Delta_{i,-1}^C = \Delta_{i,-1}^N = 0$. Using these expressions, and after some manipulation, we can express (A.72) as:

$$
\begin{aligned}
C^{\sigma-1}\mathcal{IC}_\pi =& \frac{1}{2}\left(\boldsymbol{x}_{i,t} - \boldsymbol{x}_{i,t}^*\right)^T \boldsymbol{\mathcal{H}}_0 \left(\boldsymbol{x}_{i,t} - \boldsymbol{x}_{i,t}^*\right) + \left(\boldsymbol{x}_{i,t} - \boldsymbol{x}_{i,t}^*\right)^T \boldsymbol{\mathcal{H}}_1 \left(\boldsymbol{x}_{i,t+1} - \boldsymbol{x}_{i,t+1}^*\right) \\
=& -\left\{ \frac{\sigma}{2}\left(1+\beta^{-1}\right)\widetilde{b}_{i,t}^2 + \frac{\omega_W}{2}\left(\varphi + \sigma\omega_W\right)\widetilde{n}_{i,t}^2 - \sigma\omega_W \widetilde{b}_{i,t}\widetilde{n}_{i,t} + \sigma\widetilde{b}_{i,t}\widetilde{b}_{i,t+1} - \sigma\omega_W \widetilde{b}_{i,t}\widetilde{n}_{i,t+1} \right\} \\
=& -\frac{\sigma}{2}\left(1+\beta^{-1}\right)\widetilde{b}_{i,t}^2 - \frac{\omega_W}{2}\varphi\widetilde{n}_{i,t}^2 - \frac{1}{2}\sigma\left(\Delta_{i,t}^N\right)^2 + \sigma\widetilde{b}_{i,t}\Delta_{i,t}^N - \sigma\widetilde{b}_{i,t}\widetilde{b}_{i,t+1} + \sigma\widetilde{b}_{i,t}\Delta_{i,t+1}^N \\
=& -\frac{\gamma}{2}\widetilde{c}_{i,t}^2 - \frac{\omega_W\varphi}{2}\widetilde{n}_{i,t}^2 + \frac{\gamma}{2}\widetilde{\Omega}_{i,t},
\end{aligned}
$$

[37] See Proposition 2 in the Online Appendix of Maćkowiak and Wiederholt (2015) for details.

with

$$\begin{aligned}\widetilde{\Omega}_{i,t} =& \beta^{-1}\left(\left(\Delta_{i,t}^{C}\right)^2 - \beta^{-1}\left(\Delta_{i,t-1}^{C}\right)^2\right) + \beta^{-1}\left(\left(\Delta_{i,t}^{N}\right)^2 - \beta^{-1}\left(\Delta_{i,t-1}^{N}\right)^2\right)\\ &+ \left(\Delta_{i,t}^{N}\Delta_{i,t+1}^{C} - \beta^{-1}\Delta_{i,t-1}^{N}\Delta_{i,t}^{C}\right) - \left(\Delta_{i,t}^{C}\widetilde{c}_{i,t+1} - \beta^{-1}\Delta_{i,t-1}^{C}\widetilde{c}_{i,t}\right).\end{aligned}$$

Now, note that:

$$\begin{aligned}\widetilde{\Omega}_{i,0} + \beta\widetilde{\Omega}_{i,1} &= \left(\Delta_{i,1}^{C}\right)^2 + \left(\Delta_{i,1}^{N}\right)^2 + \beta\Delta_{i,1}^{N}\Delta_{i,2}^{C} - \Delta_{i,0}^{N}\Delta_{i,1}^{C} - \beta\Delta_{i,1}^{C}\widetilde{c}_{i,2}\\ \widetilde{\Omega}_{i,0} + \beta\widetilde{\Omega}_{i,1} + \beta^2\widetilde{\Omega}_{i,2} &= \beta\left(\Delta_{i,2}^{C}\right)^2 + \beta\left(\Delta_{i,2}^{N}\right)^2 + \beta^2\Delta_{i,2}^{N}\Delta_{i,3}^{C} - \beta^2\Delta_{i,2}^{C}\widetilde{c}_{i,3}\\ &\vdots\\ \widetilde{\Omega}_{i,0} + \beta\widetilde{\Omega}_{i,1} + \ldots + \beta^T\widetilde{\Omega}_{i,T} &= \beta^{T-1}\left(\Delta_{i,2}^{C}\right)^2 + \beta^{T-1}\left(\Delta_{i,2}^{N}\right)^2 + \beta^T\Delta_{i,2}^{N}\Delta_{i,3}^{C} - \beta^T\Delta_{i,2}^{C}\widetilde{c}_{i,3}.\end{aligned}$$

It follows that:

$$\begin{aligned}\mathrm{E}_{-1}\sum_{t=0}^{\infty}\beta^t\widetilde{\Omega}_{i,t} &= \lim_{T\to\infty}\beta^{T-1}\mathrm{E}_{i,-1}\left[\left(\Delta_{i,2}^{C}\right)^2\right] + \lim_{T\to\infty}\beta^{T-1}\mathrm{E}_{i,-1}\left(\Delta_{i,2}^{N}\right)^2\\ &+ \lim_{T\to\infty}\beta^T\mathrm{E}_{i,-1}\left[\Delta_{i,2}^{N}\Delta_{i,3}^{C}\right] - \lim_{T\to\infty}\beta^T\mathrm{E}_{i,-1}\left[\Delta_{i,2}^{C}\widetilde{c}_{i,3}\right]\\ &= 0\end{aligned}$$

Consequently, the first part of (A.72) simplifies to

$$\mathcal{IC}_{\pi} = -\frac{1}{2}C^{1-\sigma}\mathrm{E}_{i,-1}\sum_{t=0}^{\infty}\beta^t\left\{\sigma\widetilde{c}_{i,t}^2 + \omega_W\varphi\widetilde{n}_{i,t}^2\right\}$$

The last step is to express $\widetilde{c}_t$ and $\widetilde{n}_t$ as a function of the information wedges. To do this, recall that equation (A.62) relates the value of current consumption for a particular household with the the prices it faces, as well as expectations about the future value of those prices. Using (A.62) and (A.64), we can express the deviations of real income from their full information counterpart as:

$$c_{i,t} - c_{i,t}^{*} = -\frac{1}{\sigma}\beta\sum_{k=0}^{\infty}\beta^k\left(\mathrm{E}_{i,t}r_{i,t+k+1}^{Z} - \mathbb{E}_t r_{i,t+k}^{Z}\right) + \beta\chi\sum_{k=0}^{\infty}\beta^k\left[\mathrm{E}_{i,t}e_{i,t+k}^{R} - \mathbb{E}_t e_{i,t+k}^{R}\right]$$

Using (A.70) and (A.71), we can express each discounted sum as:

$$\begin{aligned}\sum_{k=0}^{\infty}\beta^k\left[\mathrm{E}_{i,t}e_{i,t+k}^{R} - \mathbb{E}_t e_{i,t+k}^{R}\right] &= \sum_{k=0}^{\infty}\beta^k\nu_{i,t+k|t},\\ \sum_{k=0}^{\infty}\beta^k\left(\mathrm{E}_{i,t}r_{i,t+k+1}^{Z} - \mathbb{E}_t r_{i,t+k}^{Z}\right) &= \nu_{i,t+1|t}^{\pi} + \sum_{k=1}^{\infty}\beta^k\left\{\nu_{i,t+k+1|t}^{\pi} - \phi_{\pi}\nu_{i,t+k|t}^{\pi}\right\}.\end{aligned}$$

It follows that the deviations of consumption of household i from its full information benchmark can be written as:

$$c_{i,t} - c^*_{i,t} = -\frac{1}{\sigma}\beta\left\{\nu^{\pi}_{i,t+1|t} + \sum_{k=1}^{\infty}\beta^k\left\{\nu^{\pi}_{i,t+k+1|t} - \phi_\pi \nu^{\pi}_{i,t+k|t}\right\}\right\} + \beta\chi\sum_{k=0}^{\infty}\beta^k \nu_{i,t+k|t}.$$

Now, using the optimality condition of labor supply (14), and the observation that the nominal wage in every period is part of the household's information set, we can express the deviation of household's i labor supply from it's full-information level as

$$n_{i,t} - n^*_{i,t} = \frac{1}{\varphi}\nu_{i,t|t} - \frac{\sigma}{\varphi}\left(c_{i,t} - c^*_{i,t}\right).$$

Putting the previous results together, we have that household's i cost of not paying attention to inflation as:

$$\mathcal{IC}_\pi = -\frac{1}{2}C^{1-\sigma}\mathrm{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left\{\sigma\left(c_{i,t} - c^*_{i,t}\right)^2 + \mathcal{M}^{-1}\varphi\left(n_{i,t} - n^*_{i,t}\right)^2\right\},$$

with

$$c_{i,t} - c^*_{i,t} = -\frac{1}{\sigma}\beta\left\{\nu^{\pi}_{i,t+1|t} + \sum_{k=1}^{\infty}\beta^k\left\{\nu^{\pi}_{i,t+k+1|t} - \phi_\pi \nu^{\pi}_{i,t+k|t}\right\}\right\} + \beta\chi\sum_{k=0}^{\infty}\beta^k \nu_{i,t+k|t},$$

and

$$n_{i,t} - n^*_{i,t} = \frac{1}{\varphi}v_{i,t|t} - \frac{\sigma}{\varphi}\left(c_{i,t} - c^*_{i,t}\right).$$

A.9 Proof of Proposition 9

Following the discussion in the main text, the optimal attention problem (50) can be written as

$$\min_{\sigma^2_\epsilon} \quad \Omega \mathrm{Var}\left[\pi_t\right]\left(1 - \frac{1}{\mathrm{Var}\left[\pi_t\right] + \sigma^2_\epsilon}\right) + \omega\log\left(1 + \frac{\mathrm{Var}\left[\pi_t\right]}{\sigma^2_\epsilon}\right).$$

Define $q \equiv \mathrm{Var}\left[\pi_t\right]/\sigma^2_\epsilon$ as the signal-to-noise ratio implied by households choice of σ^2_ϵ. Since the household is atomistic, it takes $\mathrm{Var}\left[\pi_t\right]$ as given. It follows that choosing σ^2_ϵ is equivalent to choosing q, and we can restate the inattention problem as

$$\min_q \quad -\Omega\frac{q}{q+1} + \omega\log\left(1+q\right).$$

Taking first order conditions and solving for q yields

$$q = \max\left\{\frac{\Omega}{\omega} - 1, 0\right\}.$$

Now, equation 29 implies

$$1 - \psi = \frac{q}{1+q}$$

Replacing q by the optimal choice of the household yields the expression in the main text.

B First Order Conditions of Households' Problem

In this section I provide a detailed derivation of the first order conditions characterizing household's i problem. Without loss of generality, I assume $Z_{i,t} = 1$ and $D_{i,t} = 0$ in the following derivations, and drop the subscript i to keep the notation simple.

Problem Description

The problem of each household in period t is to choose C_t and N_t to maximize:

$$\mathrm{E}_t \sum_{k=0}^{\infty} \beta^k U\left(C_{t+k}, N_{t+k}\right), \tag{B.73}$$

where

$$U\left(C_t, N_t\right) \equiv \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi},$$

and C_t is a consumption index of the form

$$C_t = \left[\int_0^1 C_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{B.74}$$

Here, $P_{j,t}$ denotes the price of variety j and $P_t \equiv \left(\int_0^1 P_{j,t}^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}$ denotes the price index associated to this consumption basket. Maximization of B.73 is subject to the following budget constraint:

$$M_t + B_t = W_t N_t + Q_{t-1}^{-1} B_{t-1}, \tag{B.75}$$

where $M_t \equiv \int_0^1 P_{j,t} C_{j,t} dj$ denotes the household's total expenditures.

Recall that the relative price of each good, $P_{j,t}^R$, is part of each household's information set. We can thus solve the problem of the household in two stages. In the first stage, the household chooses the consumption level $C_{j,t}$ that minimize expected expenditures, for a given level of consumption C_t. In the second stage, the household chooses two of three variables in C_t and N_t to maximize (B.73), conditional on it's information set. At the end of the period, the household adjusts the B_t to make sure it's budget constraint (B.75) binds.

Consumption varieties. The expenditure minimization problem of each household in any period t can be written as

$$\begin{aligned} \min_{C_{j,t}} \quad & \mathrm{E}_t \left[P_t \int_0^1 P_{j,t}^R C_{j,t} d\right] \\ \text{s.t.} \quad & C_t = \left[\int_0^1 C_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}} \end{aligned}$$

The first order condition of this problem yields

$$\mathrm{E}_t\left[P_t P_{j,t}^R - \widetilde{\Lambda}_t\left(C_t/C_{j,t}\right)^{\frac{1}{\varepsilon}}\right] = 0$$

Where $\widetilde{\Lambda}_t$ is the Lagrange multiplier associated to this problem. Denote $\widehat{P}_t \equiv \mathrm{E}_t P_t$ as the belief of the household about the price level conditional on its own information set . Using the fact that $P_{j,t}^R$ is part of this information set, we can rewrite the first order condition of the household as:

$$C_{j,t} = \left(\frac{\widehat{P}_t}{\widetilde{\Lambda}_t} P_{j,t}^R\right)^{-\varepsilon} C_t. \tag{B.76}$$

Using this condition to replace $C_{j,t}$ in B.74, and using the fact that $\int_0^1 \left(P_{j,t}^R\right)^{1-\varepsilon} dj = 1$, we get:

$$\widetilde{\Lambda}_t = \widehat{P}_t$$

Using this expression to replace $\widetilde{\Lambda}_t$ back in B.76, we can express the optimal consumption of each variety as

$$C_{j,t} = \left(P_{j,t}^R\right)^{-\varepsilon} C_t \tag{B.77}$$

Conditional on this behavior, we can express the total expenditures M_t in the budget constraint B.75

$$\int_0^1 P_{j,t} C_{j,t} dj = P_t C_t. \tag{B.78}$$

Consumption and labor supply Using B.78, we can rewrite the budget constraint B.75 as

$$P_t C_t + B_t = W_t N_t + Q_{t-1}^{-1} B_{t-1}. \tag{B.79}$$

We can use the previous expression to rewrite the problem of the household in recursive form:

$$\begin{aligned} v\left(B_{t-1}\right) = \max_{C_t, N_t} \quad & \left\{U\left(C_t, N_t\right) + \beta \mathrm{E}_t\left[v\left(B_t\right)\right]\right\} \\ \text{s.t.} \quad & P_t C_t + B_t = W_t N_t + Q_{t-1}^{-1} B_{t-1} \end{aligned}$$

We can write the first order conditions of this problem as:

$$C_t^{-\sigma} = \mathrm{E}_t\left[P_t \Lambda_t\right], \tag{B.80}$$

$$N_t^{\varphi} = \mathrm{E}_t\left[W_t \Lambda_t\right], \tag{B.81}$$

$$0 = \mathrm{E}_t\left[\beta v'\left(B_t\right) - \Lambda_t\right], \tag{B.82}$$

where Λ_t is the Lagrange multiplier associated to this problem.

Now, define $\widehat{P}_t \equiv \mathrm{E}_t P_t$. We can combine (B.80) and (B.81) as

$$N_t^{\varphi} C_t^{\sigma} = \mathcal{D}_t^w \frac{W_t}{\widehat{P}_t} \tag{B.83}$$

with

$$\mathcal{D}_t^w \equiv \frac{\mathrm{E}_t\left[\widehat{P}_t \Lambda_t\right]}{\mathrm{E}_t\left[P_t \Lambda_t\right]}.$$

Note that, up to a first order approximation, $\log \mathcal{D}_t^w \approx 0$. We can thus take logs of (B.83) and subtract the corresponding expression evaluated at the non-stochastic steady-state to get :

$$\varphi n_t + \sigma c_t = w_t - \widehat{p}_t, \tag{B.84}$$

with $\widehat{p}_t \equiv \mathrm{E}_t p_t$. Now, the envelope condition of the household's problem yields:

$$\nu'\left(B_{t-1}\right) = \beta \mathrm{E}_t\left[\nu'\left(B_t\right)\right] Q_{t-1}^{-1}, \tag{B.85}$$

where I have used the fact that Q_t is part of the household's information set. Using (B.81) and B.82, we can rewrite (B.85) as

$$\nu'\left(B_{t-1}\right) = \left(\frac{N_t^{\varphi}}{W_t}\right) Q_{t-1}^{-1}. \tag{B.86}$$

Using the previous expression in (B.82) yields:

$$Q_t \frac{N_t^{\varphi}}{W_t} = \beta \mathrm{E}_t\left[\left(\frac{N_{t+1}^{\varphi}}{W_{t+1}}\right)\right]$$

Finally, using (B.83), we can rewrite the previous expression as:

$$Q_t = \beta \mathrm{E}_t\left[\frac{\mathcal{D}_{t+1}^w}{\mathcal{D}_t^w}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{\widehat{P}_t}{\widehat{P}_{t+1}}\right] \tag{B.87}$$

Taking a log-linear approximation of the previous expression yields

$$c_t = \mathrm{E}_t c_{t+1} - \frac{1}{\sigma}\left(i_t - \left(\widehat{p}_{t+1} - \widehat{p}_t\right)\right). \tag{B.88}$$

C Quantitative Model and Solution Method

In this section I present the equations characterizing the quantitative model used in Section 6 and the computational algorithm used to solve the model. To begin, I present the algorithm to compute the solution for a given level of σ_ε^2. I build on this algorithm to solve the problem under rational inattention.

C.1 Equilibrium with exogenous information

C.1.1 Equilibrium inflation and output

Each household has access to a noisy signal about the aggregate price level of the form

$$s_{i,t} = p_{i,t} + \epsilon_{i,t}; \qquad \epsilon_{i,t} \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_\varepsilon^2\right).$$

Given a precision of signals, the equilibrium levels of output and inflation are satisfy the following supply and demand relationships:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa_{PC}\left(y_t - \left((1+\varphi)/(\sigma+\varphi)\right) a_t\right) - \lambda^{-1} \nu_t \tag{C.89}$$

$$y_t = -\frac{1}{\sigma}\left(\phi \pi_t - \mathbb{E}_t \pi_{t+1} + z_{t+1} - z_t\right) + \mathbb{E}_t y_{t+1} + \mathcal{X}_t + \beta \mathbb{E}_t \mathcal{X}_{t+1} \tag{C.90}$$

where

$$\mathcal{H}_t = \chi \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \nu_{t+k},$$

$$\mathcal{R}_t = -\sigma^{-1} \mathbb{E}_t \left\{ \nu^{\pi}_{t+1|t} + \sum_{k=1}^{\infty} \beta^k \left\{ \nu^{\pi}_{t+k+1|t} - \phi_\pi \nu^{\pi}_{t+k|t} \right\} \right\},$$

$$\chi \equiv \left(\frac{1-\beta}{\beta}\right)\left(\frac{\mathcal{M}\varphi}{\mathcal{M}\varphi + \sigma}\right),$$

where $\nu_t \equiv \int_0^1 \{p_t - \mathrm{E}_{i,t} p_t\}\, dj$, $\nu^{\pi}_{t+k+1|t} \equiv \int_0^1 \{\pi_{t+k+1} - \mathrm{E}_{i,t}\pi_{t+k+1}\}\, dj$, and $z_t$and a_t denote the aggregate demand and technology shocks, which are given by

$$z_t = \rho_{AD} z_{t-1} + \eta_t^{AD}; \quad \eta_t^{AD} \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_{AD}^2\right)$$

$$a_t = \rho_{TFP} a_{t-1} + \eta_t^{TFP}; \quad \eta_t^{TFP} \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_{TFP}^2\right)$$

C.1.2 Beliefs

To compute the solution of this model note that, by Wold's representation theorem, any equilibrium π_t has finite second moments allows for an $MA(\infty)$ representation of both variables. Since shocks hitting the economy are causal, we can invert this polynomial to get a $AR(\infty)$ representation for π_t. We can then approximate numerically the law of motion of π_t to an arbitrary degree of accuracy by a finite-lag $AR(H)$ process.

Let $\boldsymbol{\pi}_t = \left(\pi_t, \pi_{t-1}, \ldots, \pi_{t-(H-1)}\right)'$ represent a vectors stacking current and $H-1$ lags of the π_t, and denote as $\boldsymbol{e}_i$ is the i-th column of the identity matrix. We can write the reduced-form $AR(H)$ of π_t in state-space form as[38]

$$\boldsymbol{\pi}_t = \boldsymbol{\Phi}_\pi \boldsymbol{\pi}_{t-1} + \boldsymbol{e}_1 \left(\underbrace{\psi_\pi^{TFP} \eta_t^{TFP} + \psi_\pi^{AD} \eta_t^{AD}}_{u_t} \right). \tag{C.91}$$

The $H \times H$ matrix $\boldsymbol{\Phi}_\pi$, together with the impact coefficients $\left(\psi_\pi^{TFP}, \psi_\pi^{AD}\right)$, summarize the behavior of inflation and are equilibrium objects to be determined[39]. We can use (C.91) to derive an $AR(H)$ process for p_t of the form

$$\boldsymbol{p}_t = \boldsymbol{\Phi}_A \boldsymbol{p}_{t-1} + \boldsymbol{e}_1 u_t, \tag{C.92}$$

with $\boldsymbol{p}_t = (p_t, p_{t-1}, \ldots, p_{t-H})'$. Equations

Since, $\mathrm{E}_{i,t}\left[\varepsilon_{i,t}\varepsilon_{k,t}\right] = 0$ for all $i \in [0,1]$ and $k \neq i$, we can characterize the beliefs about each relative price for each household independently using (28) and (C.92). Using Assumption 1 and standard Kalman filter formulas[40] yields

$$\widehat{\boldsymbol{p}}_{i,t|t} = \widehat{\boldsymbol{p}}_{i,t|t-1} + \boldsymbol{K}_A \boldsymbol{e}_1' \left(\boldsymbol{p}_{j,t} - \widehat{\boldsymbol{p}}_{i,t|t-1}\right) + \boldsymbol{K}_A \boldsymbol{e}_1' \varepsilon_{i,t}, \tag{C.93}$$

with $\widehat{\boldsymbol{p}}_{i,t|s} = \mathrm{E}_{i,s}\left[\boldsymbol{p}_t\right]$. The Kalman gain vector $\boldsymbol{K}_A$ is a $H \times 1$ vector given by

$$\boldsymbol{K}_A = \left(\frac{1}{\widehat{\boldsymbol{\Sigma}}_A[1,1] + \sigma_\epsilon^2}\right) \widehat{\boldsymbol{\Sigma}}_A \boldsymbol{e}_1,$$

where $\boldsymbol{\Sigma}_A[1,1]$ denotes the $[1,1]$ element of the covariance matrix $\widehat{\boldsymbol{\Sigma}}_A \equiv \mathrm{Var}_{i,t-1}\left[\boldsymbol{p}_t\right]$. This matrix can be found by solving the following Algebraic Riccati equation

$$\widehat{\boldsymbol{\Sigma}}_A = \boldsymbol{\Phi}_A \widehat{\boldsymbol{\Sigma}}_A \boldsymbol{\Phi}_A' - \left(\widehat{\boldsymbol{\Sigma}}_A[1,1] + \sigma_\epsilon^2\right)^{-1} \boldsymbol{\Phi}_A \widehat{\boldsymbol{\Sigma}}_A \boldsymbol{e} \boldsymbol{e}_1' \widehat{\boldsymbol{\Sigma}}_A \boldsymbol{\Phi}_A' + \sigma_u^2 \boldsymbol{e}_1 \boldsymbol{e}_1',$$

with $\sigma_u^2 \equiv \mathrm{Var}\left[u_t\right]$. Notice that this matrix is constant and common across households as consequence of Assumptions 1 and 2.

Now let $\widehat{\boldsymbol{\pi}}_{t|s} \equiv \int_0^1 \mathrm{E}\left[\boldsymbol{p}_t - \boldsymbol{p}_{t-1} | \mathcal{I}_{i,s}\right] di$ denote the average belief across households about

[38] See Chapter 3 in Hamilton (1994).

[39] Stability of the process implies that all eigenvalues of $\boldsymbol{\Phi}_\pi$. Notice, however, that $\boldsymbol{\Phi}_A$ may have an eigenvalue equal to 1.

[40] See Durbin and Koopman (2012).

the inflation rate. Let $\boldsymbol{L}_H$ denote the $H \times H$ shift matrix

$$\boldsymbol{L}_H \equiv \begin{bmatrix} 0 & 0 & & \cdots & 0 \\ 1 & 0 & & & \vdots \\ \vdots & 1 & \ddots & & \\ & & \ddots & 0 & 0 \\ 0 & & \cdots & 1 & 0 \end{bmatrix}$$

and let $\boldsymbol{D}_H \equiv \boldsymbol{I}_H - \boldsymbol{L}_H$. Premultiplying both sides of (C.93) by $\boldsymbol{D}_H$ yields

$$\widehat{\boldsymbol{\pi}}_{i,t|t} = \widehat{\boldsymbol{\pi}}_{i,t|t-1} + \boldsymbol{K}_\pi \boldsymbol{e}_1' \left(\boldsymbol{p}_{j,t} - \widehat{\boldsymbol{p}}^A_{i,t|t-1} \right) + \boldsymbol{K}_\pi \boldsymbol{e}_1' \varepsilon_{i,t}, \tag{C.94}$$

with $\boldsymbol{K}_\pi \equiv \boldsymbol{D}_H \boldsymbol{K}_A$ denoting a $H \times 1$ vector of Kalman gains for inflation beliefs. Moreover, let $\nu_{i,t,s} \equiv p_{j,t,} - \widehat{p}_{i,j,t|s}$ denote each household forecast error about aggregate price in period t, conditional on her own information set up to period s. Writing (C.92) one period ahead and subtracting the corresponding forecast by the household using (C.93) yields

$$\boldsymbol{\nu}_{i,t+1|t} = \boldsymbol{\Phi}_A \boldsymbol{\nu}_{i,t|t} + \boldsymbol{e}_1 u_{t+1}$$

Subtracting $\boldsymbol{p_t}$ from (C.92) and manipulating terms, we get

$$\boldsymbol{\nu}_{i,t|t} = \left(\boldsymbol{I}_H - \boldsymbol{K}_A \boldsymbol{e}_1' \right) \boldsymbol{\nu}_{i,t|t-1} - \boldsymbol{K}_A \boldsymbol{e}_1' \varepsilon_{i,t}$$

Putting these two expressions together, we arrive to

$$\boldsymbol{\nu}_{i,t|t} = \Psi_A \boldsymbol{\nu}_{i,t-1|t-1} + \boldsymbol{\delta}_A u_t - \boldsymbol{K}_A \boldsymbol{e}_1' \varepsilon_{i,t}$$

with $\Psi_A \equiv \left(\boldsymbol{I}_H - \boldsymbol{K}_A \boldsymbol{e}_1' \right) \boldsymbol{\Phi}_A$ and $\boldsymbol{\delta}_A \equiv (\boldsymbol{e}_1 - \boldsymbol{K}_A)$. Notice that this implies that forecast errors are a combination of surprises in aggregate demand an household-specific idiosyncratic noise

$$\boldsymbol{\nu}_{i,t+1|t} = \boldsymbol{\Phi}_A \Psi_A \boldsymbol{\Phi}_A^{-1} \boldsymbol{\nu}_{i,t|t-1} + \boldsymbol{e}_1 \psi_\pi u_{t+1} - \boldsymbol{\Phi}_A \boldsymbol{K}_A \boldsymbol{e}_1' \varepsilon_{i,t} \tag{C.95}$$

Following similar steps, we can derive an analogous representation for the forecast errors about each relative price. For the inflation rate, recall that equation (C.92) has an associated representation for the inflation rate

$$\boldsymbol{\pi}_{t+1} = \boldsymbol{\Phi}_\pi \boldsymbol{\pi}_t + \boldsymbol{e}_1 \psi_\pi u_{t+1}$$

Subtracting the household forecast of the aggregate inflation rate yields

$$\boldsymbol{\nu}^\pi_{i,t+1|t} = \boldsymbol{\Phi}_\pi \boldsymbol{\nu}^\pi_{i,t|t} + \boldsymbol{e}_1 u_{t+1}$$

Subtracting π_t from (C.94) yields

$$\boldsymbol{\nu}^{\pi}_{i,t|t} = \boldsymbol{\nu}^{\pi}_{i,t|t-1} - \boldsymbol{K}_{\pi}\boldsymbol{e}_1'\boldsymbol{\nu}_{i,t|t-1} - \boldsymbol{K}_{\pi}\boldsymbol{e}_1'\varepsilon_{i,t}$$

Replacing in the previous equation and using the results for $\boldsymbol{\nu}_{i,t|t}$, we get

$$\boldsymbol{\nu}^{\pi}_{i,t|t} = \boldsymbol{\Phi}_{\pi}\boldsymbol{\nu}^{\pi}_{i,t-1|t-1} + \boldsymbol{\delta}_{\pi}u_t - \boldsymbol{\Upsilon}_{\pi}\boldsymbol{\nu}_{i,t-1|t-1} - \boldsymbol{K}_{\pi}\boldsymbol{e}_1'\varepsilon^{A}_{i,t} \tag{C.96}$$

with $\boldsymbol{\delta}_{\pi} \equiv (\boldsymbol{e}_1 - \boldsymbol{K}_{\pi})$ and $\boldsymbol{\Upsilon}_{\pi} = \boldsymbol{K}_{\pi}\boldsymbol{e}_{H1}'\boldsymbol{\Phi}_A$.
The previous expressions imply that households perception and forecast errors display persistence over time, from the perspective of a fully informed agent that observes these errors externally. These beliefs are dispersed due to the idiosyncratic shopping experiences of each household. But the average belief still displays persistence over time due to learning. Let $\boldsymbol{x}_t \equiv \int_0^1 \boldsymbol{x}_{i,t}di$ denote the average belief across households of a vector of variables $\boldsymbol{x}_t$. Equations (C.93) and (C.94) imply that the average belief about the aggregate price level and the inflation rate follow

$$\widehat{\boldsymbol{p}}_{t|t} = \widehat{\boldsymbol{p}}_{t|t-1} + \boldsymbol{K}_A\boldsymbol{e}_1'\left(\boldsymbol{p}_t - \widehat{\boldsymbol{p}}_{t|t-1}\right)$$

$$\widehat{\boldsymbol{\pi}}_{t|t} = \widehat{\boldsymbol{\pi}}_{t|t-1} + \boldsymbol{K}_{\pi}\boldsymbol{e}_1'\left(\boldsymbol{p}_t - \widehat{\boldsymbol{p}}_{t|t-1}\right)$$

Furthermore, we can integrate equations (C.96), (C.95), across households to get the following expressions for the average perception error about inflation and the price level:

$$\boldsymbol{\nu}^{\pi}_{t|t} = \boldsymbol{\Phi}_{\pi}\boldsymbol{\nu}^{\pi}_{t-1|t-1} + \boldsymbol{\delta}_{\pi}\eta^{z}_t - \boldsymbol{K}_{\pi}\boldsymbol{e}_1'\boldsymbol{\Phi}_A\boldsymbol{\nu}^{P}_{t-1|t-1} \tag{C.97}$$

$$\boldsymbol{\nu}_{t|t} = \Psi_A\boldsymbol{\nu}_{t-1|t-1} + \boldsymbol{\delta}_A u_t \tag{C.98}$$

To conclude, notice that this characterization implies

$$\mathbb{E}_t\boldsymbol{\nu}_{t+k|t} = \boldsymbol{\Phi}^{k}_{A}\boldsymbol{\nu}_{t|t}$$

$$\mathbb{E}_t\boldsymbol{\nu}_{t+k|t+k} = \Psi^{A}_{A}\boldsymbol{\nu}_{t|t}$$

Similarly

$$\mathbb{E}_t\boldsymbol{\nu}^{\pi}_{t+k|t} = \boldsymbol{\Phi}^{k}_{\pi}\boldsymbol{\nu}^{\pi}_{t|t}$$
$$\mathbb{E}_t\boldsymbol{\nu}^{\pi}_{t+k|t+k} = \boldsymbol{\Phi}^{k}_{\pi}\boldsymbol{\nu}^{\pi}_{t|t} - \boldsymbol{\Pi}_k\boldsymbol{\nu}^{P}_{t|t}$$

with $\boldsymbol{\Pi}_k = \sum_{j=1}^{k}\boldsymbol{\Phi}_{\pi}^{k-j}\boldsymbol{\Upsilon}_{\pi}\boldsymbol{\Psi}_{A}^{j-1}$.

C.2 Solution

To compute the solution of the model, with exogenous information, I use the following algorithm:

1. Guess $\Theta \equiv \left(\mathbf{\Phi}_{\pi}, \psi_{\pi}^{TFP}, \psi_{\pi}^{TFP}\right)$ using the corresponding solution under full-information.

2. Use (C.97) and (C.98) to express y_t and π_t in (C.89) and (C.90) as a function of $\boldsymbol{\nu}_{t|t}$, $\widehat{\boldsymbol{\pi}}_{t|t}$ and $\boldsymbol{u}_t$ only.

3. Find the ARMA process associated to the SS representation implied by (C.97) and (C.98) and the expressions obtained in the previous step (See Chapter 12 in Brockwell and Davis (2009))

4. Find the AR process representation of the previous ARMA process, truncated to H lags.

5. Update Θ based on the previos AR representation and go back to step (2) until convergence.